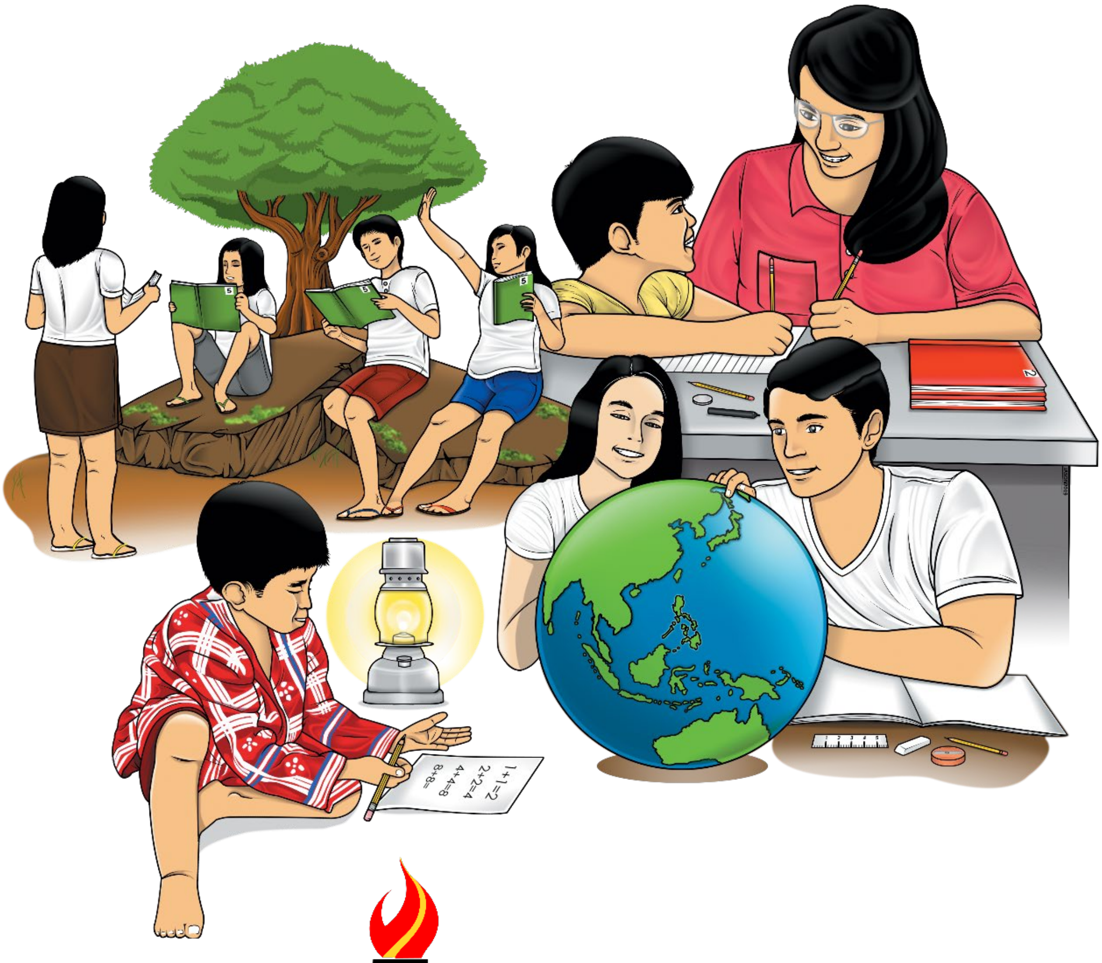
Senior High School

General Mathematics

Quarter 1 – Module 6:
Solving Rational Equations and Inequalities



ALTERNATIVE DELIVERY MODE

General Mathematics
Alternative Delivery Mode
Quarter 1 – Module 6: Solving Rational Equations and Inequalities
First Edition, 2021

Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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Printed in the Philippines by ________________________

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Solving Rational Equations and Inequalities

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written for learners like you to determine a method and set of steps for solving rational equations and inequalities. Learners like you can also explore and develop new methods that you have synthesized and apply these techniques for performing operations with rational expressions.

In this module, you will able to explain the appropriate methods in solving rational equations and inequalities you used. You will also be able to check and explain extraneous solutions.

After going through this module, you are expected to:

1. Apply appropriate methods in solving rational equations and inequalities.
2. Solve rational equations and inequalities using algebraic techniques for simplifying and manipulating of expressions.
3. Determine whether the solutions found are acceptable for the problem by checking the solutions.

What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following expressions is an equality between two expressions containing one or more variables?
    a. rational function
    b. rational equation
    c. rational inequality
    d. irrational expression

2. What do you call a root obtained in the process of solving an equation which looks correct but after analyzing it turns out as incorrect?
    a. extraneous solution
    b. rational expression
    c. least common denominator
    d. quotient

3. What do you call an inequality which involves one or more rational expressions?
    a. rational function
    b. rational equation
    c. rational inequality
    d. irrational expression
4. What is the usual technique to a solve rational equation?
    a. multiply both sides of the equation by its greatest common factor
    b. multiply both sides of the equation by its least common denominator
    c. multiply both sides of the equation by its inverse factor
    d. multiply both sides of the equation by its greatest common denominator

For items 5-9: Refer to the rational equation below.

$$\frac{x}{3}+\frac{1}{4}=\frac{x}{2}$$

5. What is the LCD of the denominator 3, 4 and 2?
    a. 3
    b. 6
    c. 8
    d. 12

6. What property will be used if you multiply the LCD on both sides of the equation?
   a. Distributive Property
   b. Associative Property
   c. Commutative Property
   d. Additive Property

7. What will be the new form of the equation after applying the property and simplifying?
   a. $4x + 3 = 6x$
   b. $3x + 4 = 2x$
   c. $6x + 4 = 3x$
   d. $12x + 3 = 12x$

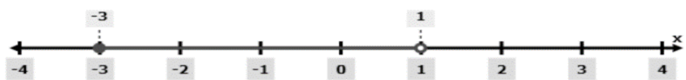
8. What will be the solution on the given rational equation?
   a. $\frac{2}{3}$
   b. $\frac{3}{2}$
   c. 2
   d. 3

9. How will you check if your solution is correct?
   a. by eliminating the rational expressions.
   b. by dividing both sides of the equation by LCD.
   c. by applying Commutative Property.
   d. by substituting the answer or solution in the original equation.

10. What will you if you obtained a solution that made the expression in the equation undefined?
   a. Accept even if it is untrue value.
   b. Do not reject since it will satisfy the equation in a long run.
   c. Discard the solution since it is unreal.
   d. Continue the solution even if it will give undefined answer.

11. Which of the following is NOT an inequality sign?
   a. $\leq$
   b. $\sqrt{}$
   c. $\geq$
   d. $<$

12. Express the graph of solution set into interval notation.



   a. $\{x \mid -3 \leq x < 1\}$
   b. $\{x \mid -3 \leq x \leq 1\}$
   c. $\{x \mid \ \ 3 < x \leq 1\}$
   d. $\{x \mid \ \ 3 \leq x < 1\}$

13. Below are the steps in solving rational inequality EXCEPT
    a. Put the inequality in general form.
    b. Set the numerator and denominator equal to one and solve.
    c. Plot the critical values on a number line, breaking the number line into intervals and take a test number from each interval by substituting into the original inequality.
    d. Determine if the endpoints of the intervals in the solution should be included in the intervals.

14. Solve for the solutions of the rational inequality $\frac{(x+3)}{(x-2)} \leq 1$.
    a. [∞, 2)
    b. (∞, 2]
    c. (-∞, 2)
    d. [-∞, 2)

15. How will you know that the critical points for item no. 14 will satisfy the inequality?
    a. If it makes a true statement, then the interval from which it came from is not in the solution.
    b. If it makes a false statement, then the interval from which it came from is in the solution.
    c. If it makes a true statement, then the interval from which it came from is in the solution.
    d. If it makes a false statement, then the interval from which it came from is either in the solution or not.

Lesson 1 Solving Rational Equations and Inequalities

In this lesson, you shall explore more about solving rational equations and inequalities by carefully studying the step by step methods of solutions. You will first start from the easiest procedures in solving this type of equation and as you progress, you will learn more techniques and concepts that will help you to solve more complex problems related to this topic. Exercises will range from the simplest problems to the most complex.

At this point, students like you have already solved a variety of equations, including linear and quadratic equations from the previous grade level. Rational equations and inequalities follow the sequence of solving problems by combining the concepts used in solving both linear and quadratic equations. Students will be assessed using both formative and summative assessments along the way to best evaluate your progress.

What's In

Let's Review!

How do you solve algebraic expressions? What are the different properties you need to apply to solve problems involving rational equations and inequalities?

For you to begin, you need to recall some properties and processes to simplify rational expressions by answering the following problems below. Write your answer inside the box.

1. Simplify the given rational expression: $\frac{x-2}{x^2-4}$

2. Multiply the given rational expressions: $\frac{3x+1}{x^2-1} \cdot \frac{x+1}{3x^2+x}$

3. Find the sum of given rational expressions with like denominators: $\frac{5x-1}{x-8} + \frac{3x+4}{x-8}$

4. Find the difference of the given rational expressions with unlike denominators:

$$\frac{6}{x^2-4} - \frac{2}{x^2-5x+6}$$

Let's check if you have made it! You can also write your solution on the prepared box to compare the techniques you apply.

1. To simplify the rational expression you can do the following steps.

Steps in simplifying rational expression	$\frac{x-2}{x^2-4}$	
1. Factor the denominator of the rational expression.	$\frac{x-2}{(x-2)(x+2)}$	Write your previous solution here for comparison.
2. Cancel the common factor.	$\frac{\cancel{x-2}}{(\cancel{x-2})(x+2)}$	
3. Write the simplified rational expression.	$\frac{1}{x+2}$	

2. To multiply rational expressions you can do the following steps.

Steps in multiplying rational expressions	$\frac{3x+1}{x^2-1} \cdot \frac{x+1}{3x^2+x}$	
1. Factor out all possible common factors.	$\frac{3x+1}{(x+1)(x-1)} \cdot \frac{x+1}{x(3x+1)}$	Write your previous solution here for comparison.
2. Multiply the numerators and denominators.	$\frac{(3x+1)(x+1)}{(x+1)(x-1)(x)(3x+1)}$	
3. Cancel out all common factors.	$\frac{(\cancel{3x+1})(\cancel{x+1})}{(\cancel{x+1})(x-1)(x)(\cancel{3x+1})}$	
4. Write the simplified rational expression.	$\frac{1}{x(x-1)}$	

3. To add and subtract rational expressions with like denominators you can do the following steps.

Steps in addition or subtraction of rational expressions with like denominators	$\frac{5x-1}{x-8}+\frac{3x+4}{x-8}$	
1. Add or subtract the numerators of both expressions and keeping the common denominator.	$\frac{5x-1+3x+4}{x-8}$	Write your previous solution here for comparison.
2. Combine like terms in the numerator.	$\frac{5x+3x+4-1}{x-8}$	
3. Write the simplified rational expression.	$\frac{8x+3}{x-8}$	

4. To add and subtract rational expressions with unlike denominators you can do the following steps.

Steps in adding or subtracting rational expressions with unlike denominators	$\frac{6}{x^2-4}-\frac{2}{x^2-5x+6}$	
1. Factor the denominator of each fraction to help find the LCD.	$\frac{6}{(x-2)(x+2)}-\frac{2}{(x-2)(x-3)}$	Write your previous solution here for comparison.
2. Find the least common denominator (LCD).	$LCD: (x-2)(x+2)(x-3)$	
3. Multiply each expression by its LCD	$\frac{6(LCD)}{(x-2)(x+2)}-\frac{2(LCD)}{(x-2)(x-3)}$	
4. Write the simplified expression.	$6(x-3)-2(x+2)$	
5. Let the simplified expression as the numerator and the LCD as the denominator of the new fraction	$\frac{6x-18-2x-4}{(x-2)(x+2)(x-3)}$	

6. Combine like terms and reduce the rational expression if you can. In this case, the rational expression cannot be simplified.	$\frac{4x-22}{(x-2)(x+2)(x-3)}$	

How was the activity? Did you answer all the reviewed items correctly? Great! If you did, then you can now move forward on the next stage of this topic and I am confident that it will be very easy for you to understand the lesson.

Notes to the Teacher

Please remind our students that learning mathematics is a linear process wherein the math skills and knowledge from the previous modules and grade level will be used throughout this topic. For example, if the students have not mastered arithmetic properties and processes then they will have difficulty with the current topic because it requires all of these prerequisite skills. Therefore, it will be necessary to go back, review previous topics and problem-solving skill before they can continue. Inspire our students that learning is not always onward and upward, sometimes we have to take a glimpse of the past before we can move forward.

What's New

Follow Me Activity

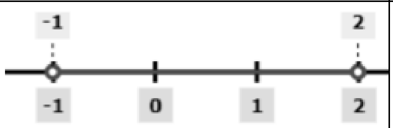
Solving Rational Equations and Inequalities

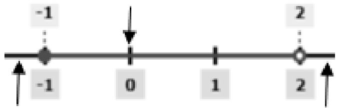
Before you proceed on the lesson proper try to answer the rational equation and inequality using guided procedure. You can synthesize your own steps in solving the problem. You can refer to previous activities if you are having difficulty processing arithmetic properties. Hope you enjoy answering before you continue to the next part of the discussion.

1. Solve example 2 of the rational equation by following the given steps.

	Example 1	Example 2
Rational Equation	$\frac{x-3}{x^2-25}+\frac{1}{x+5}=\frac{1}{(x-5)}$	$\frac{2}{x^2-1}-\frac{1}{x-1}=\frac{1}{2}$
1. Find the Least Common Denominator (LCD).	LCD: $(x+5)(x-5)$	
2. Multiply both sides of the equation by its the LCD.	$(x+5)(x-5)[\frac{x-3}{x^2-25}+\frac{1}{x+5}=\frac{1}{(x-5)}]$	
3. Apply the Distributive Property and then simplify.	$(x-3)+1(x-5)=1(x+5)$ $x-3+x-5=x+5$ simplify: $2x-8=x+5$ $2x-x=8+5$ $x=13$	
4. Find all the possible values of x.	$x=13$	
5. Check each value by substituting into original equation and reject any extraneous root/s	Checking: $\frac{x-3}{x^2-25}+\frac{1}{x+5}=\frac{1}{(x-5)}$ $\frac{13-3}{13^2-25}+\frac{1}{13+5}=\frac{1}{(13-5)}$ $\frac{10}{169-25}+\frac{1}{18}=\frac{1}{8}$ $\frac{10}{144}+\frac{1}{18}=\frac{1}{8}$ $\frac{10+8}{144}=\frac{1}{8}$ $\frac{1}{8}=\frac{1}{8}$ ✓ Note: No extraneous root	

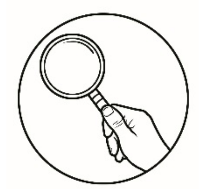
2. Solve example 2 of rational inequality. You can refer to example 1 for the guided steps.

	Example 1	Example 2
Rational Inequality	$\frac{3}{x-2} \leq -1$	$\frac{3x+1}{x-1} \geq 2$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)} > 0$ where > can be replaced by $<, \leq$ *and* $\geq$	$\frac{3}{x-2} + 1 \leq 0$	
2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	$\frac{3+1(x-2)}{x-2} \leq 0$ $\frac{x+1}{x-2} \leq 0$	
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	Numerator: $x+1=0$ $x=-1$ Denominator: $x-2=0$ $x=2$	
4. Plot the critical values on a number line, breaking the number line into intervals.		
5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.	$\frac{3}{x-2} \leq -1$ when $x=-1$ $\frac{3}{-1-2} \leq -1$ $\frac{3}{-3} \leq -1$ $-1 \leq -1$ ✓ ($x=-1$ is included in the solution) when $x=2$ $\frac{3}{2-2} \leq -1$ $\frac{3}{0} \leq -1$ $undefined \leq -1$ ✗ ($x=2$ is not included in the solution)	

5. Select test values in each interval and substitute those values into the inequality. Note: If the test value makes the inequality true, then the entire interval is a solution to the inequality. If the test value makes the inequality false, then the entire interval is not a solution to the inequality.	 when $x = -2$ $\frac{3}{-2-2} \leq -1$ $\frac{3}{-4} \leq -1$ ✗ $false$ when $x = 0$ $\frac{3}{0-2} \leq -1$ $\frac{3}{-2} \leq -1$ ✓$true$ when $x = 3$ $\frac{3}{3-2} \leq -1$ $3 \leq -1$ ✗ $false$	
6. Use interval notation or set notation to write the final answer.	$[-1,2)$	

How do you find the activity? Have you enjoyed it? Did you follow the steps correctly? The activity tells you about solving rational equations and inequalities. Yes, you read it right. You almost got it!

Let's check if your answers are correct and which process did you find it difficult. I hope you enjoyed answering by your own.



What is It

Rational equation is an equation containing at least one rational expression with a polynomial in the numerator and denominator. It can be used to solve a variety of problems that involve rates, times and work. Using rational expressions and equations it can help us to answer questions about how to combine workers or machines to complete a job on schedule.

Let us use the previous activity to discuss and deepen your knowledge and skills in solving rational equation. The first thing to be in your mind in solving rational equation is to eliminate all the fractions.

Let us solve

$$\frac{2}{x^2-1}-\frac{1}{x-1}=\frac{1}{2}$$

Step 1. You need to find the Least Common Denominator (LCD).
The LCD of the given fractions is $2(x-1)(x+1)$
Step 2. You need to multiply LCD to both sides of the equation to eliminate the fractions. You can also apply cross multiplication if and only if you have one fraction equal to one fraction, that is, if the fractions are proportional. In this case you cannot use the cross multiplication unless you simplify the left equation into a single fraction.

$$2(x-1)(x+1)\left[\frac{2}{x^2-1}-\frac{1}{x-1}=\frac{1}{2}\right]$$

Step 3. You simplify the resulting equation using the distributive property and then combine all like terms.

$$2(2)-2(x+1)=(x-1)(x+1)$$
$$4-2x-2=x^2-1$$
$$x^2+2x-3=0$$

Step 4. You need to solve the simplified equation to find the value/s of x. In this case, we need to get the equation equal to zero and solve by factoring.

$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x+3=0 \ or \ x-1=0$$
$$x=-3 \ or \ x=1$$

So possible solutions are -3 and 1.
Step 5. Finally, you can now check each solution by substituting in the original equation and reject any extraneous root/s (which do not satisfy the equation).

$$\frac{2}{x^2-1}-\frac{1}{x-1}=\frac{1}{2}$$

When $x=-3$

$$\frac{2}{(-3)^2-1}-\frac{1}{(-3)-1}=\frac{1}{2}$$
$$\frac{2}{8}+\frac{1}{4}=\frac{1}{2}$$
$$\frac{1}{2}=\frac{1}{2} \quad \checkmark$$

When $x=1$

$$\frac{2}{(1)^2-1}-\frac{1}{(1)-1}=\frac{1}{2}$$
$$\frac{2}{0}-\frac{1}{0}=\frac{1}{2}$$
$$0=\frac{1}{2} \quad \times$$

In this case, $x=-3$ is the only solution. That's why it is always important to check all solutions in the original equations. You may find that they yield untrue statements or produce undefined expressions.

Rational inequality is an inequality which contains one or more rational expressions. It can be used in engineering and production quality assurance as well as in businesses to control inventory, plan production lines, produce pricing models, and for shipping/warehousing goods and materials.

Solving an inequality is much like solving a rational equation except that there are additional steps that focus on illustrating the solution set of an inequality on a number line.

Let us use problem number 2 in the previous activity to discuss and deepen your knowledge and skills in solving rational inequality.

$$\frac{3x+1}{x-1} \geq 2$$

Step 1. Put the rational inequality in the general form where > can be replaced by $<,\ \leq and\ \geq$.

$$\frac{R(x)}{Q(x)} > 0$$

$$\frac{3x+1}{x-1} - 2 \geq 0$$

Step 2. Write the inequality into a single rational expression on the left-hand side.

$$\frac{3x+1-2(x-1)}{x-1} \geq 0$$

$$\frac{x+3}{x-1} \geq 0$$

Note: Remember that one side must always be zero and the other side is always a single fraction, so simplify the fractions if there is more than one fraction.

Step 3. Set the numerator and denominator equal to zero and solve. The values you get are called **critical values**.

Numerator: $x+3=0$

$x=-3$

Denominator: $x-1=0$

$x=1$

Step 4. Plot the critical values on a number line, breaking the number line into intervals.

Step 5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.

$$\frac{3x+1}{x-1} \geq 2$$

when $x = -3$

$$\frac{3(-3)+1}{(-3)-1} \geq 2$$

$$\frac{-8}{-4} \geq 2$$

$$2 \ \geq 2 \quad \checkmark \,(\, x = -3 \text{ is included in the solution})$$

when $x = 1$

$$\frac{3(1)+1}{(1)-1} \geq 2$$

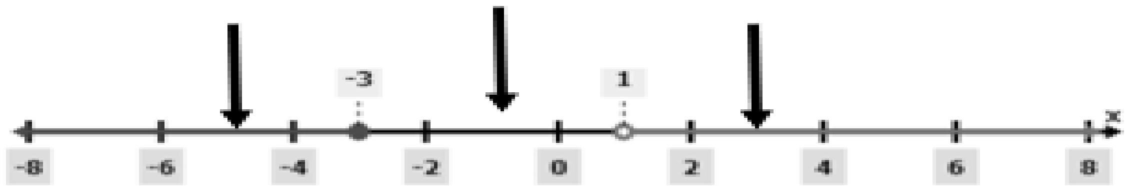
$$\frac{4}{0} \geq 2$$

$$undefined \ \geq 2 \quad X \,(\, x = 1 \text{ is not included in the solution})$$

See the illustration below.

Step 6. Select test values in each interval and substitute those values into the inequality.

$$\frac{3x+1}{x-1} \geq 2$$



when $x = -5$

$$\frac{3(-5)+1}{(-5)-1} \geq 2$$

$$\frac{-14}{-6} \geq 2$$

$$\frac{7}{3} \ or \ 2.33 \ \geq 2 \ (\, x = -5 \text{ TRUE})$$

when $x = -1$

$$\frac{3(-1)+1}{(-1)-1} \geq 2$$

$$\frac{-2}{-2} \geq 2$$

$$1 \geq 2 \quad (x = -1 \text{ FALSE})$$

when $x = 3$

$$\frac{3(3)+1}{(3)-1} \geq 2$$

$$\frac{10}{2} \geq 2$$

$$5 \geq 2 \quad (x = 5 \text{ TRUE})$$

Note:

a. If the test value makes the inequality **TRUE**, then the entire interval is a **solution** to the inequality.

b. If the test value makes the inequality **FALSE**, then the entire interval is **not a solution** to the inequality.

Step 7. Use interval notation to write the final answer.

$$\boxed{(-\infty, -3] \cup (1, \infty)}$$

Let's learn more!

Solve each rational equation and inequality.

1. $\frac{4x+1}{x+1} - 3 = \frac{12}{x^2-1}$

2. $\frac{2x-8}{x-2} \geq 0$

Solution:

Rational Equation	$\frac{4x+1}{x+1} - 3 = \frac{12}{x^2-1}$
1. Find the Least Common Denominator (LCD).	LCD: $(x+1)(x-1)$
2. Multiply both sides of the equation by its the LCD.	$(x+1)(x-1)[\frac{4x+1}{x+1} - 3 = \frac{12}{x^2-1}]$
3. Apply the Distributive Property and then simplify.	$(x-1)(4x+1) - 3(x+1)(x-1) = 12$ simplify: $4x^2 - 3x - 1 - 3x^2 + 3 = 12$

	$x^2 - 3x + 2 = 12$ $x^2 - 3x - 10 = 0$ Factor $(x-5)(x+2) = 0$
4. Find all the possible values of x.	$x - 5 = 0$ $\quad x = 5$ $x + 2 = 0$ $\quad x = -2$
5. Check each value by substituting into original equation and reject any extraneous root/s	Checking: $\frac{4x+1}{x+1} - 3 = \frac{12}{x^2-1}$ when $x = 5$ $\frac{4(5)+1}{5+1} - 3 = \frac{12}{5^2-1}$ $\frac{21}{6} - 3 = \frac{12}{24}$ $\frac{3}{6} = \frac{12}{24}$ $\frac{1}{2} = \frac{1}{2}$ ✓ when $x = -2$ $\frac{4(-2)+1}{(-2)+1} - 3 = \frac{12}{(-2)^2-1}$ $\frac{-7}{-1} - 3 = \frac{12}{3}$ $4 = 4$ ✓ Note: No extraneous root

Rational Inequality	$\frac{2x-8}{x-2} \geq 0$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)} > 0$ where > can be replaced by $<, \leq$ *and* $\geq$	This inequality is already in general form. We are all set to go.
2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	This inequality is already in a single rational expression wherein 0 is on one side. $\frac{2x-8}{x-2} \geq 0$
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	Numerator: $2x - 8 = 0$ $2x = 8$ $x = 4$

<table>
<tr>
<td></td>
<td>Denominator:
$x - 2 = 0$
$x = 2$</td>
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<tr>
<td>4. Plot the critical values on a number line, breaking the number line into intervals.</td>
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</td>
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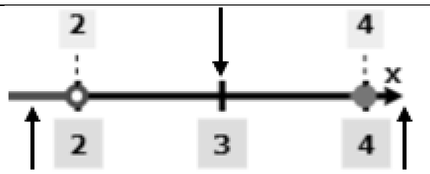
<tr>
<td>5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.</td>
<td>$\frac{2x-8}{x-2} \geq 0$

when $x = 2$
$\frac{2(2)-8}{2-2} \geq 0$
$\frac{-4}{0} \geq 0$
$undefined \geq 0$ ✗
($x = 2$ is not included in the solution)

when $x = 4$
$\frac{2(4)-8}{4-2} \geq 0$
$\frac{0}{2} \geq 0$
$0 \geq 0$ ✓
($x = 4$ is included in the solution)</td>
</tr>
<tr>
<td>6. Select test values in each interval and substitute those values into the inequality.
Note:
If the test value makes the inequality true, then the entire interval is a solution to the inequality.
If the test value makes the inequality false, then the entire interval is not a solution to the inequality.</td>
<td>

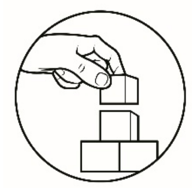
$\frac{2x-8}{x-2} \geq 0$

when $x = 1$
$\frac{2(1)-8}{1-2} \geq 0$
$\frac{-6}{-1} \geq 0$
$6 \geq 0$ ✓ *true*

when $x = 3$
$\frac{2(3)-8}{3-2} \geq 0$
$\frac{-2}{1} \geq 0$
$-2 \geq 0$ ✗ *false*

when $x = 5$</td>
</tr>
</table>

	$\frac{2(5)-8}{5-2} \geq 0$ $\frac{2}{3} \geq 0 \quad \checkmark \; true$
7. Use interval notation or set notation to write the final answer.	$(-\infty, 2) \cup [4, \infty)$



What's More

Activity 1.

Solve the following rational equations and inequalities using the guided procedure on the table below.

1.

Rational Equation	$\frac{x-2}{x^2-4} + \frac{1}{x+2} = \frac{1}{x-2}$
1. Find the Least Common Denominator (LCD).	
2. Multiply both sides of the equation by its the LCD.	
3. Apply the Distributive Property and then simplify.	
4. Find all the possible values of x.	$x = 6$
5. Check each value by substituting into the original equation and reject any extraneous root/s	

2.

Rational Equation	$\frac{x-6}{x^2-4x-12}+\frac{2}{x+2}=\frac{1}{x-6}$
1. Find the Least Common Denominator (LCD).	
2. Multiply both sides of the equation by its the LCD.	
3. Apply the Distributive Property and then simplify.	
4. Find all the possible values of x.	$x = 10$
5. Check each value by substituting into original equation and reject any extraneous root/s	

3.

Rational Inequality	$\frac{2(x-4)}{x}<-4$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)}>0$ where > can be replaced by $<, \leq$ *and* $\geq$	
2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	
4. Plot the critical values on a number line, breaking the number line into intervals.	
5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.	

6. Select test values in each interval and substitute those values into the inequality. Note: If the test value makes the inequality true, then the entire interval is a solution to the inequality. If the test value makes the inequality false, then the entire interval is not a solution to the inequality.	
7. Use interval notation or set notation to write the final answer.	$(0, \frac{3}{4})$

4.

Rational Inequality	$\frac{x^2+x-6}{x^2-3x-4} \leq 0$
1. Put the rational inequality in general form. $\frac{R(x)}{Q(x)} > 0$ where > can be replaced by $<, \leq$ and $\geq$	
2. Write the inequality into a single rational expression on the left side. (You can refer to the review section for solving unlike denominators)	
3. Set the numerator and denominator equal to zero and solve. The values you get are called critical values.	
4. Plot the critical values on a number line, breaking the number line into intervals.	
5. Substitute critical values to the inequality to determine if the endpoints of the intervals in the solution should be included or not.	
6. Select test values in each interval and substitute those values into the inequality. Note: If the test value makes the inequality true, then the entire interval is a solution to the inequality. If the test value makes the inequality false, then the entire interval is not a solution to the inequality.	
7. Use interval notation or set notation to write the final answer.	$[-3, -1) \cup [2, 4)$

Activity 2

Solve each problem below and choose the letter that corresponds to the solution to each problem. Place the correct answer in the corresponding lines.

What did the bible verses 1John 4:7-21 is all about?

______ ______ ______   ______ ______   ______ ______ ______ ______

1 2 3   4 5   6 7 8 9

1. $\frac{x+2}{3} = \frac{2x-4}{2}$
2. $\frac{7}{4x} - \frac{3}{x^2} = \frac{1}{2x^2}$
3. $\frac{2x}{x+1} + \frac{5}{2x} = 2$
4. $\frac{x^2-1}{x-3} = \frac{8}{x-3}$
5. $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2-8x+12}$
6. $\frac{5x}{x-1} < 4$
7. $\frac{x}{x-2} - 7 = \frac{2}{x-2}$
8. $\frac{x^2+x-12}{x-1} \leq 0$
9. $\frac{3x+1}{x-2} \geq 5$

A. -3
C. -1 and 6
D. -5
E. $(2, \frac{11}{2}]$
G. 4
I. 3
L. (-4, 1)
N. -3 and 3
O. 2
S. -1
V. $(-\infty, -4] \cup (1, 3]$
Y. $(-\infty, -4) \cup [1, 3)$

What I Have Learned

Complete the following statements by writing the correct word or words and formulas.

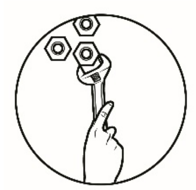
1. A ________________________ is an equation containing at least one rational expression with a polynomial in the numerator and denominator.
2. To determine if the endpoints of the intervals in the solution should be included or not you need to _____________________ the critical values to the inequality.
3. In order to get the critical values you need to set __________________________ and _________________________ equal to zero.
4. The first step in solving rational inequality is to put the inequality in general form where in one side must always be ____________________________ and the other side is in a _________________________ fraction.

5. If the test value makes the inequality ________________________, then the entire interval is a solution to the inequality.

Topics for Discussion

1. Explain the similarities and differences in solving between rational equations and inequalities?

2. Why do extraneous solutions sometimes occur and don't work in the original form of the equation?



What I Can Do

Read the situation carefully and solve the problem.

The new COVID-19 testing facility in Lucena City is operating with two laboratory technicians. Technician A takes 2 hours to finish 50 samples of specimens from CoVID-19 patients. Technician B takes 3 hours to finish 45 samples of specimens from COVID-19 patients. Working together, how long should it take them to finish 150 samples of specimens from COVID-19 patients?

Hint:
Think about how many samples of specimens each technician can finish in one hour. This is their testing rate.

Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. It is an equation containing at least one fraction whose numerator and denominator are polynomials.
   a. rational function
   b. rational equation
   c. rational inequality
   d. irrational equation

2. The usual technique to eliminate denominator in solving a rational equation is to multiply both sides of the equation by its ________________.
   a. inverse factor
   b. greatest common factor
   c. least common denominator
   d. greatest common denominator

3. An inequality which involves one or more rational expressions is called ________________.
   a. rational function
   b. rational equation
   c. rational inequality
   d. irrational equation

4. You can only use cross multiplication in solving rational equation if and only if you have one fraction equal to one fraction, that is, if the fractions are ___________________________.
   a. negative
   b. positive
   c. inequal
   d. proportional

5. If the test value makes the inequality _________________________, then the entire interval is not a solution to the inequality.
   a. true
   b. false
   c. proportional
   d. reciprocal

For items 6-7: Refer to the rational equation below.

$$\frac{5}{2x-4}+\frac{2}{x+3}=\frac{3}{x-2}$$

6. To solve the equation, we multiply both sides by
   a. $x-2$
   b. $x+3$
   c. $(x+2)(x-3)$
   d. $(x-2)(x+3)$

7. Which of the following will be the solution to the given rational equation?
   a. $\frac{11}{3}$
   b. $\frac{3}{11}$
   c. $-\frac{11}{3}$
   d. $-\frac{3}{11}$

For items 8-10: Refer to the rational inequality below.

$$\frac{x+12}{x+2}\leq 2$$

8. What are the critical values in the given rational inequality?
   a. $x\ =\ -2\ and\ 8$
   b. $x\ =-2\ and\ -8$
   c. $x\ =-2\ and\ 12$
   d. $x\ =-2\ and\ -12$

9. Which of the critical value or values is/are included as endpoints of the intervals?
   a. -2
   b. 2
   c. -8
   d. 8

10. Which of the following is the solution in the given inequality?
   a. $(-\infty,-2)\cup(8,\infty)$
   b. $(-\infty,-2]\cup[8,\infty)$
   c. $(-\infty,-2)\cup[8,\infty)$
   d. $(-\infty,-2)\cup[-8,\infty)$

For items 11-13, solve for the solutions of the following rational equations.

11. $\frac{2}{x+2} + \frac{1}{x-2} = \frac{3}{x}$
    a. 6
    b. -6
    c. 8
    d. -8

12. $\frac{8}{x^2} + 1 = \frac{9}{x}$
    a. $-1\ and -8$
    b. $1\ and\ 8$
    c. $-1\ and\ 8$
    d. $1\ and -8$

13. $\frac{1}{x^2} - 16 = 0$
    a. ± 1
    b. ± 2
    c. $\pm \frac{1}{2}$
    d. $\pm \frac{1}{4}$

For items 14-15, solve for the solutions of the following rational equations.

14. $\frac{5}{x-3} > \frac{3}{x+1}$
    a. $(-\infty, -7) \cup (-1, \infty)$
    b. $(-\infty, -1) \cup (3, \infty)$
    c. $(-7, -1) \cup (3, \infty)$
    d. $(-7, -1] \cup [3, \infty)$

15. $\frac{(x-3)(x+2)}{x-1} \leq 0$
    a. $(-\infty, -2) \cup (1,3]$
    b. $(-\infty, -2] \cup (1,3]$
    c. $(-\infty, -2] \cup [1,3)$
    d. $(-\infty, -2) \cup [1,3)$

Additional Activities

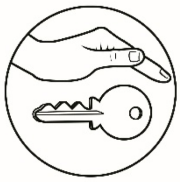
Practice Worksheet: Solving Rational Equations and Inequalities

Solve each equation. Check extraneous solutions for rational equations. Write your answer in interval notation for rational inequalities.

LEVEL 1	
1. $\dfrac{8}{x+1} = \dfrac{4}{3}$	2. $2x + 3 = \dfrac{x}{4}$
3. $\dfrac{x-4}{x+5} \leq 0$	4. $\dfrac{x+3}{3x-6} > 0$
5. $\dfrac{4}{x} + \dfrac{1}{3x} = 9$	6. $\dfrac{1}{x^2-4} \leq 0$

LEVEL 2	
7. $$\frac{20}{x}-\frac{20}{x-2}=\frac{4}{x}$$	8. $$\frac{2}{x^2-x}=\frac{1}{x-1}$$
9. $$\frac{x-9}{3x+2}\geq 3$$	10. $$\frac{x+32}{x+6}\leq 6$$
11. $$\frac{4}{x}+\frac{1}{x^2}=\frac{1}{5x^2}$$	12. $$1+\frac{2}{x+1}<\frac{2}{x}$$

LEVEL 3	
13. $$\frac{3x}{x+1}=\frac{12}{x^2-1}+2$$	14. $$\frac{2}{4x^2-9}+\frac{1}{2x-3}=\frac{3}{2x+3}$$
15. $$\frac{(x^2+1)(x-2)}{(x-1)(x+1)}\geq 0$$	16. $$\frac{(x+7)(x-3)}{(x-5)^2}>0$$
17. $$\frac{x-3}{2x+10}+2x-12=\frac{x^2+3x-18}{2x+10}$$	18. $$\frac{12x^3+16x^2-3x-4}{8x^3+12x^2+10x+15}<0$$



Answer Key

What I Know

1. B
2. A
3. C
4. B
5. D
6. A
7. A
8. B
9. D
10. C
11. B
12. A
13. B
14. D
15. C

What's More

1. G
2. O
3. D
4. I
5. S
6. L
7. O
8. V
9. E

What I Have Learned

1. Extraneous solutions
2. Substitute
3. Numerator, denominator
4. zero, single
5. true

What I Can Do

It will take 3 hours and 45 minutes for the two laboratory technicians to finish 150 samples of specimens.

Assessment

1. B
2. C
3. C
4. D
5. B
6. D
7. A
8. A
9. D
10. C
11. A
12. B
13. D
14. C
15. A

Additional Activities

1. $x = 5$
2. $x = -\frac{12}{7}$
3. $(-5,4]$
4. $(-\infty,-3) \cup (3,\infty)$
5. $x = \frac{13}{27}$
6. $(-2,2)$
7. $x = -8$
8. $x = 2$
9. $\left[-\frac{13}{5}, -\frac{2}{3}\right)$
10. $(-\infty,-6) \cup [-\frac{4}{5},\infty)$
11. $x = -\frac{1}{5}$
12. $(-2,-1) \cup (0,1)$
13. $x = -2 \text{ and } 5$
14. $x = \frac{7}{2}$
15. $(-1,1) \cup [2,\infty)$
16. $(-\infty,-7) \cup (3,5) \cup (5,\infty)$
17. $x = 7$
18. $\left(-\frac{3}{2},-\frac{4}{3}\right) \cup \left(-\frac{1}{2},\frac{1}{2}\right)$

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