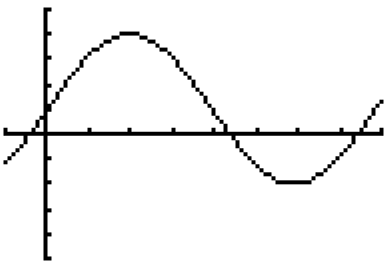
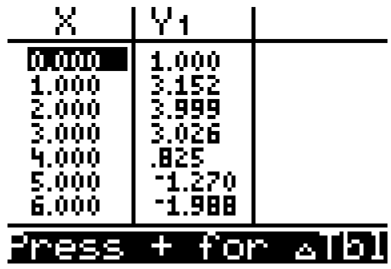
Particle motion describes the physics of an object (a point) that moves along a line; usually horizontal. There are 3 different functions that model this motion.

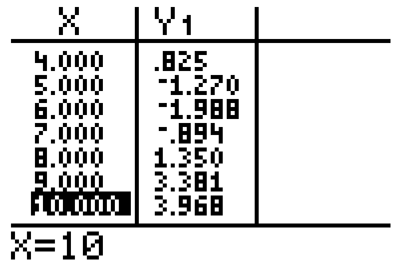
1. The position function, s(t), which describes the position of the particle along the line. As an example, consider the function, $s(t) = 3\sin(.8t) + 1$ and it's accompanying graph and table below.





X	Y1
0.000	1.000
1.000	3.152
2.000	3.999
3.000	3.026
4.000	.825
5.000	-1.270
6.000	-1.988

Press + for ΔTbl



X	Y1
4.000	.825
5.000	-1.270
6.000	-1.988
7.000	-.894
8.000	1.350
9.000	3.381
10.000	3.968

X=10

The x-values in the table, and on the graph, represents time; t = 0 is the starting time, positive t values are future times while negative t values represent going back in time. The y-values represent the position of the particle on the number line. In the table above, the particle is located at 1 when t = 0 and at 3.025 when t = 3 Both positions are to the right of zero. At t = 5, the particle is considered to be at -1.270 which is to the left of zero.

The particle is considered moving to the right when the position function is increasing.
The particle is considered moving to the left when the position function is decreasing.
The particle is considered not moving at all relative extrema, and in fact, is changing directions.

The two ways to determine the location of the relative extrema is to take the derivative of s(t) - which is the velocity function, v(t) – and set it equal to zero and solve for t. Using a graphing calculator, you can use the 'maximum' or 'minimum' programs found in the CALC menu. Using this program with the function above, the rel max occurs at (1.963, 4) and the rel min occurs at (5.89, -2). This means the particle changes directions when t = 1.963, 5.89. The y-values describe where on the number line the particle is located when the direction change occurs.

The total distance traveled (TDT) by the particle over a certain time interval [a, b], is the sum of all the absolute y-value differences between the endpoints and any relative extrema on [a, b].

Example: Find the total distance traveled by the particle on [0, 8] – which is the graph above.

TDT = $|s(0) - s(1.963)| + |s(1.963) - s(5.89)| + |s(5.98) - s(8)| = 3 + 6 + 3.350 = 12.35$

TDT uses the y-values of the endpoints and the relative extrema. TDT has to be positive.

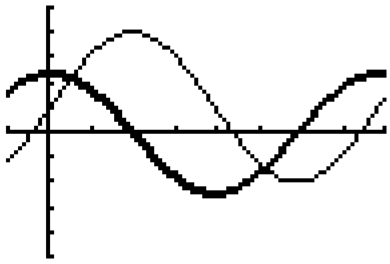
The total displacement of the particle over the interval [a,b] is simply: TD = s(b) – s(a).

$TD = s(8) - s(0) = 1.350 - 1 = 0.350$ TD can be negative!

The absolute maximum of s(t) on a closed interval represents the farthest right the particle travels to on the interval. The absolute minimum of s(t) on a closed interval represents the farthest left the particle travels to on the interval.

Therefore, on the interval [0,10] – see tables above – the furthest right the particle travels to is 4, while the farthest left the particle travels to it -2.

2. The velocity function, v(t), is the first derivative of the position function, s(t). Conversely, the position function is the first anti-derivative of position. Using the example of the derivative of s(t) above, $v(t) = s'(t) = 3\cos(0.8t) * (0.8) = 2.4\cos(0.8t)$, this uses the chain rule and the constant multiple rule. The graph (bold) and tables of v(t) are shown below:



X	Y1	Y2
0.000	1.000	2.400
1.000	3.152	1.672
2.000	3.999	-.070
3.000	3.026	-1.770
4.000	.825	-2.396
5.000	-1.270	-1.569
6.000	-1.988	.210

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X	Y1	Y2
4.000	.825	-2.396
5.000	-1.270	-1.569
6.000	-1.988	.210
7.000	-.894	1.861
8.000	1.350	2.384
9.000	3.381	1.460
10.000	3.968	-.349

X=10

The y-values of v(t) represent the velocity (how fast its moving). The particle is considered moving to the right when the velocity function is positive (above the x-axis). The particle is considered moving to the left when the velocity function is negative (below the x-axis). The particle is considered not moving at all x-intercepts, and in fact, is changing directions when v(t) changes signs at the x-intercepts.

*Notice that the x-intercepts of the velocity function correspond to the relative extrema of the position function. This true of any function and its derivative. The table below shows this:

X	Y1	Y2
0.000	1.000	2.400
1.963	4.000	9.5E-4
5.890	-2.000	-9E-4
8.000	1.350	2.384

X=

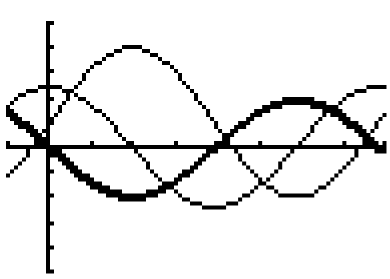
Y_1 represents s(t) and Y_2 represents v(t). The middle two values are the relative extrema of s(t) and x-intercepts of v(t). The middle Y_2 values are essentially zero (scientific notation).

When a function is increasing, it's derivative is positive and when a function is decreasing, it's derivative is decreasing. Since the derivative of velocity will be acceleration, you can surmise when v(t) is increasing, the acceleration of the particle is positive. The relative extrema of a function means it's derivative is equal to zero, so the relative extremas of v(t) mean s(t) = 0, or the particle is not accelerating.

The particle is moving the fastest when the y-values of v(t) are farthest from zero. The particle is moving the slowest when the y-values of v(t) are closest to zero. The sign (+ or –) makes no difference in the determination of 'speed'.

3. The acceleration function, a(t), is the first derivative of velocity and the second derivative of position. With the example in these notes:

$s(t) = 3\sin(0.8t) + 1$, $v(t) = 2.4\cos(0.8t)$, $a(t) = -1.92\sin(0.8t)$ The graph and tables of a(t) are below.



X	Y3	
0.000	0.000	
1.000	-1.377	
2.000	-1.919	
3.000	-1.297	
4.000	.112	
5.000	1.453	
6.000	1.913	

Press + for ΔTbl

The y-values represent the acceleration of the particle at different times.

The particle is speeding up when the velocity and acceleration of the particle both have the same sign on some interval of time. The particle is slowing down when the velocity and acceleration both have opposite signs on some interval of time.

There are a couple of different ways to determine the signs of velocity and acceleration:

1. V(t) > 0 when v(t) is above the x-axis or when s(t) is increasing.
2. V(t) < 0 when v(t) is below the x-axis or when s(t) is decreasing.
3. A(t) > 0 when a(t) is above the axis, v(t) is increasing, or when s(t) is concave up.
4. A(t) < 0 when a(t) is below the axis, v(t) is decreasing, or when s(t) is concave down.

Is it possible, and required for the test, to determine when a particle is speeding up or slowing down from JUST the position function or the velocity function using the above criteria!!

The position graph changes concavity when a(t) = 0, thus to find the point of inflection of s(t), you need to set a(t) = 0 and solve for t, or find the zeros of the a(t) function using the 'zero' program under the CALC menu.

By using the program on the graph of a(t) below, the zeros of a(t) on [0,10] are located at:
t = {0, 3.927, 7.854]   You will need to know these time values (points of inflection) if determining speeding up/slowing down from the s(t) function! Otherwise, you can't exactly determine concavity.