SYMMETRY AND TESSELLATIONS

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Teacher Bulletin
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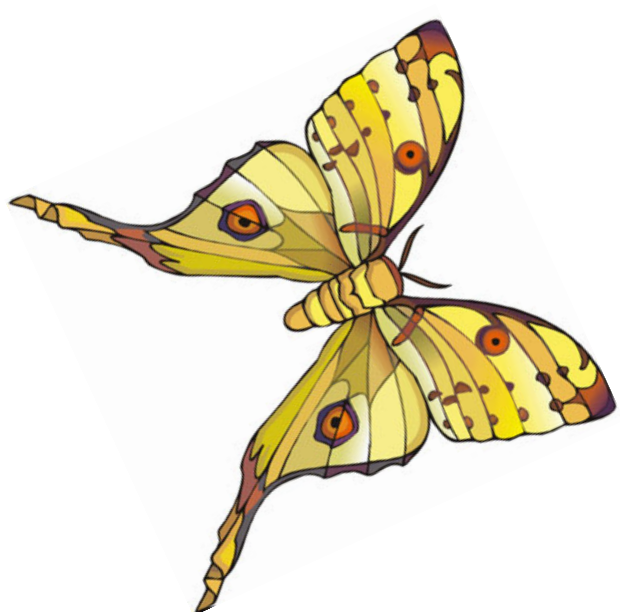

Lesson Flow of This Unit

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Round and Round the Vertex: Naming a Pattern (Portfolio)

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Tessellations
Tessellations and Polygons
Color Makes a Difference (Portfolio)
Types of Symmetry

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Reflections and Flips

ReplicatorTiles Strike! (Portfolio)
Tessellating With Irregular Polygons

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Portfolio: Your own irregular triangle tessellation.
Who Was M.C.Escher?

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Your Own Escher Tessellation

MPEG-4: Get Inspired With Escher .m4v
Templates – Your own Escher tessellation(Portfolio)
WEBQUEST: Your Company Tiles a Floor

(also available in lesson form)

Worksheet
Additional Projects: Finding Math in the World

Media Resources

Name and File Format	Name of Resource	Brief Description	
TB13 Welcome to Symmetry and Tessellations.mp4 Movie	***Welcome to Symmetry and Tessellations***	Intro and welcome for landing page	
TB13 Nature's Geometry: Symmetry and Polygons in Nature.pptx PowerPoint	***Nature's Geometry: Symmetry and Polygons in Nature.***	Brief explanation of the symmetries and polygons and illustrated in nature	
TB13 Intro to Tessellations and Definitions.pptx PowerPoint	***Intro to Tessellations and Definitions***	Defines tessellation and the definitions needed to work with them.	
TB13 M.C.Escher's Tessellations.pptx PowerPoint	***M.C.Escher's Tessellations***	Brief description of Escher tessellations	
TB13 Your Own Escher Tessellation.pptx PowerPoint	***Your Own Escher Tessellation***	Step by step of instructions and pictures to show you how to create an Escher-like tessellation.	
TB13 Get Inspired With Escher.m4v MPEG-4	***Get Inspired With Escher***	M.C.Escher's best known tessellations set to music. To view for pleasure and inspiration	
WEBQUEST: Your Company Tiles a Floor	***Your Company Tiles a Floor***	Students form a flooring company and flooring tile and bid a job.	
TB13 How To Teach This Unit.mp4 MPEG-4	***How To Teach This Unit***	Hints for teaching this unit.	
TB13 Using A Premade Template for Tessellations.mp4 MPEG-4	***Using A Premade Template for Tessellations***	Instructions and recommendations for making different pieces of art from the same template.	

Symmetry and Tessellations:

Where Math Meets Art

Symmetry occurs in nature and has been endlessly copied by man. Tessellations are used in many areas of our lives, but few of us know the term. **A tessellation is simply a repeated shape that forms a pattern and covers a plane area.** Creating tessellations is a fun and painless way to learn many geometric terms and concepts, as well as creating some very intriguing art.

In this unit we will play with some polygons, learn relationships within patterns, explore symmetries, and end with tessellations. Along the way, we will create a portfolio of art, including some like the famous M. C. Escher created.

Questions that you will answer:

1. What are polygons?
2. What shapes will tessellate?
3. Why will certain shapes tessellate while others will not?
4. What is symmetry?
5. How many different tessellating patterns can we create using two or more regular polygons?
6. Do tessellating designs have symmetry?
7. What are transformations?
8. How can we use transformations (slides/translations, flips/reflections, and turns/rotations) to create unique tessellations?
9. Are all symmetries tessellations?
10. Who is Escher and where can examples of his work be found?
11. How can Escher-like art be created?
12. What programs are available for creating tessellations online?

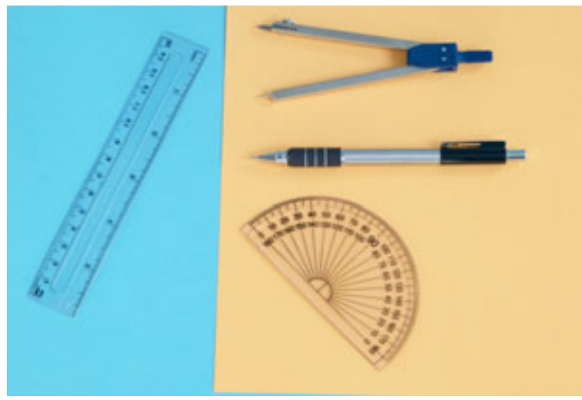
TEACHERS READ THIS FIRST

If you are not willing to do the hands-on material, don't do this unit.

If you ARE willing to do it, then here is what you need. Get it together in a large tub. It is a lot less frustrating if each student has his/her own set of supplies.

The List

1. **pattern blocks or paper pattern blocks. There are templates in this unit to make your own paper pattern blocks. If you can, print them on different colors of paper.**
2. **glue sticks**
3. **something to color with—crayons, markers, paint, or colored pencils**
4. **scissors**
5. **ruler & pencil & eraser**
6. **graph paper**
7. **construction paper**
8. **compass**
9. **protractor**
10. **lots of white paper**



Pre-Unit Experimentation. You need to play with these programs NOW, before you start teaching. Clicking on the link in the pdf will take you directly there.

Dynamic Paper

http://illuminations.nctm.org/ActivityDetail.aspx?ID=205

Need a pentagonal pyramid that's six inches tall? Or a number line that goes from (-18) to 32 by 5's? Or a set of pattern blocks where all shapes have one-inch sides? You can create all those things and more with the Dynamic Paper tool. Place the images where you want, then export it as a PDF activity sheet for your students or as a JPEG image for use in other applications or on the web.

Mirror Tool

http://illuminations.nctm.org/ActivityDetail.aspx?ID=24

This tool is used to experiment with symmetry.

Tessellations Creator

http://illuminations.nctm.org/ActivityDetail.aspx?ID=202

This program will let you create and print out tessellations

Inspirational Thought

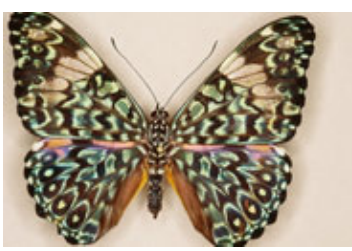
Psalm 19
Amplified Bible (AMP)

1 The heavens declare the glory of God; and the firmament shows and proclaims His handiwork.
2 Day after day pours forth speech, and night after night shows forth knowledge.

It is easy to be awed when looking up at a clear night sky. The incredible array of stars, the glowing moon, all are on such a vast scale. The advances of the Hubble telescope opened up an even larger expanse. Man discovered that far more was out there in space than he had ever imagined. Galaxies of immense proportions swirl, and different light spectrums showed spectacular patterns and complexities. Recently it has been discovered that the stars do sing. Some of the stars' songs have even been posted on YouTube so all can enjoy them. http://youtu.be/Hd_iK6IMHCE

At the same time, progress has been made in seeing the tiny world too. To the surprise of many, the microscopic world is just as complex and extensive. A person could focus a lifetime of study on one little organism, and never feel as if everything about that organism was discovered.

And on that small level, many of the same patterns emerge. Some of these patterns include rotational, reflective, and bilateral symmetry, and tessellations.



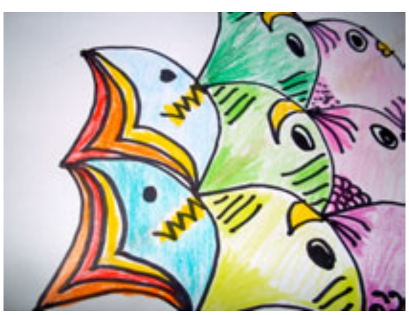
Once aware of these definitions, these things begin to pop. Suddenly you will see the bilateral symmetry not only in a butterfly but in a frog. In a still pond you will see the reflective symmetry. A sunflower shows its rotational symmetry as its seeds tessellate. These mathematical concepts show up again and again.

As you learn more about symmetry and tessellations, it is my prayer that you will see them in the world around you. May you always be conscious of the great God who originated all these wonderful patterns, and interpreted them in such an infinite number of ways.

This page contains the work that it is possible to have in the Portfolio. Please check or modify the pieces you would like depending on grade and time available

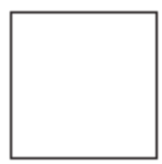
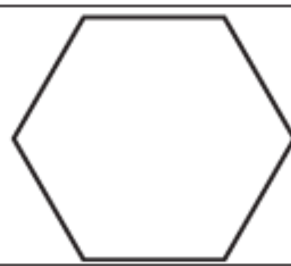
Portfolio of_________________

1. Closed Figures
2. Angle Measurement
3. Interior Angles
4. Polygons Make Great Tessellations
5. Round and Round the Vertex: Naming a Pattern
6. Patterns of Polygons
7. Color Makes A Difference
8. Alphabet Reflective Symmetry
9. Example of Symmetry
10. ReplicatorTiles Strike!
11. Tessellating With Irregular Polygons: Add color to two squares
12. Tessellating With Irregular Polygons: Your own irregular triangle tessellation.
13. Templates—your own Escher tessellation
14. WebQuest results: Your Company Tiles a Floor



Printable Resource Page:

Pattern Blocks Vocabulary Cards

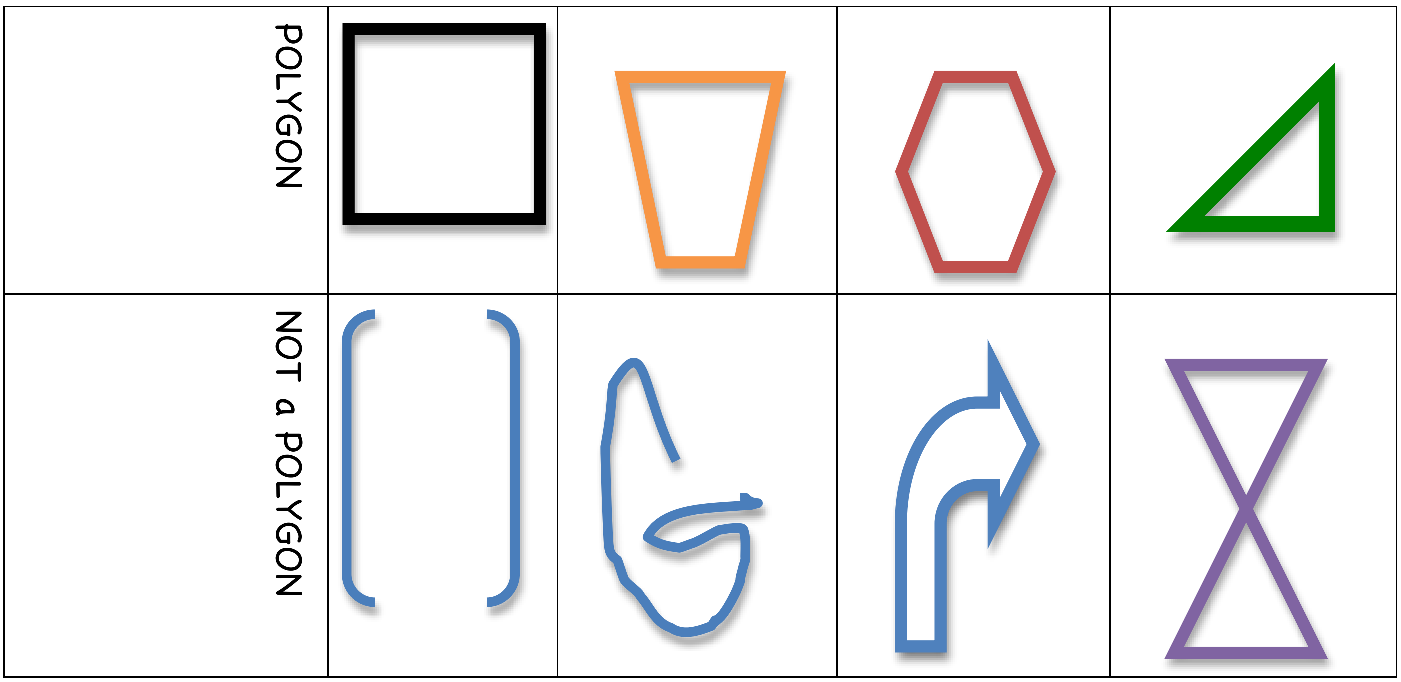
trapezoid	
rhombus	
triangle	
square	
hexagon	
parallelogram	

What Is A Polygon?

Start by presenting the PowerPoint "Nature's Geometry: Symmetry and Polygons in Nature." This will be an overview of what the unit will work with.

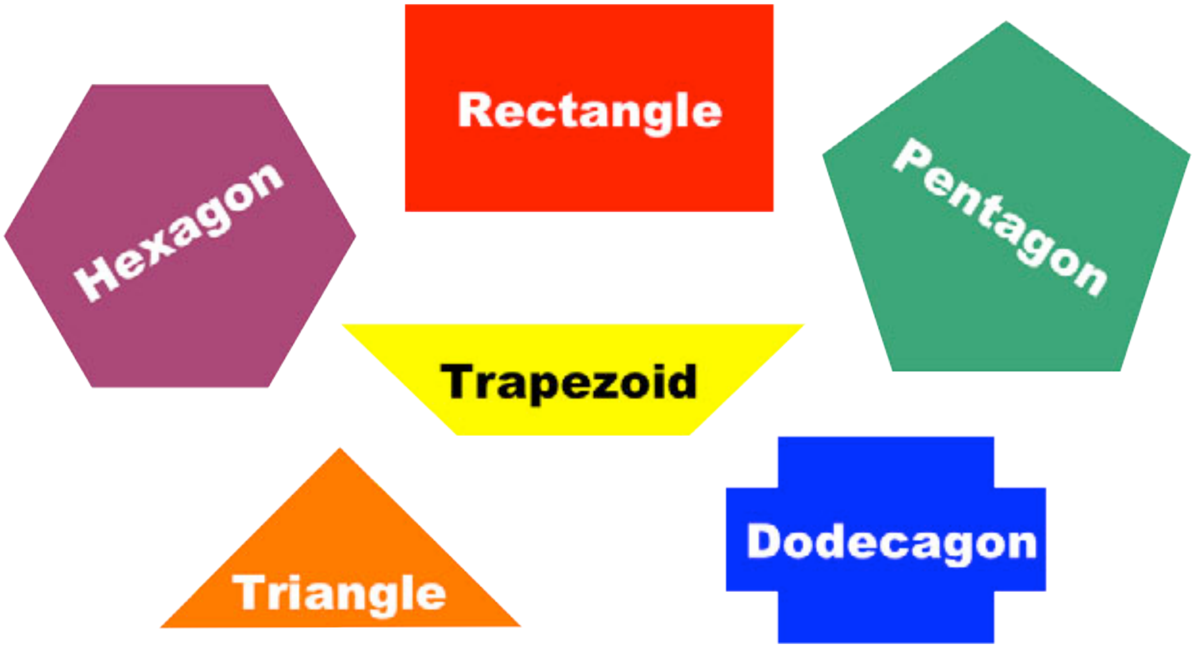
It is best to start with a review of polygons. There is a unit in the Teacher Bulletin by Martha Ban which can be used. However, if you use the terms as you work with the shapes, students will pick them up very rapidly.

Students should know that a polygon is a closed shape made of line segments, and that it can be regular or irregular. Geometry terms such as angle, line segments, and vertex are also helpful to know.



Use the **Dynamic Paper Tool** to experiment with different polygons, or to print some that are unique to your situation. http://illuminations.nctm.org/ActivityDetail.aspx?ID=205

To begin, name a few of the basic polygons:



Great Online Resources

Dynamic Paper
http://illuminations.nctm.org/ActivityDetail.aspx?ID=205
Need a pentagonal pyramid that's six inches tall? Or a number line that goes from -18 to 32 by 5's? Or a set of pattern blocks where all shapes have one-inch sides? You can create all those things and more with the Dynamic Paper tool. Place the images you want, then export it as a PDF activity sheet for your students or as a JPEG image for use in other applications or on the web.

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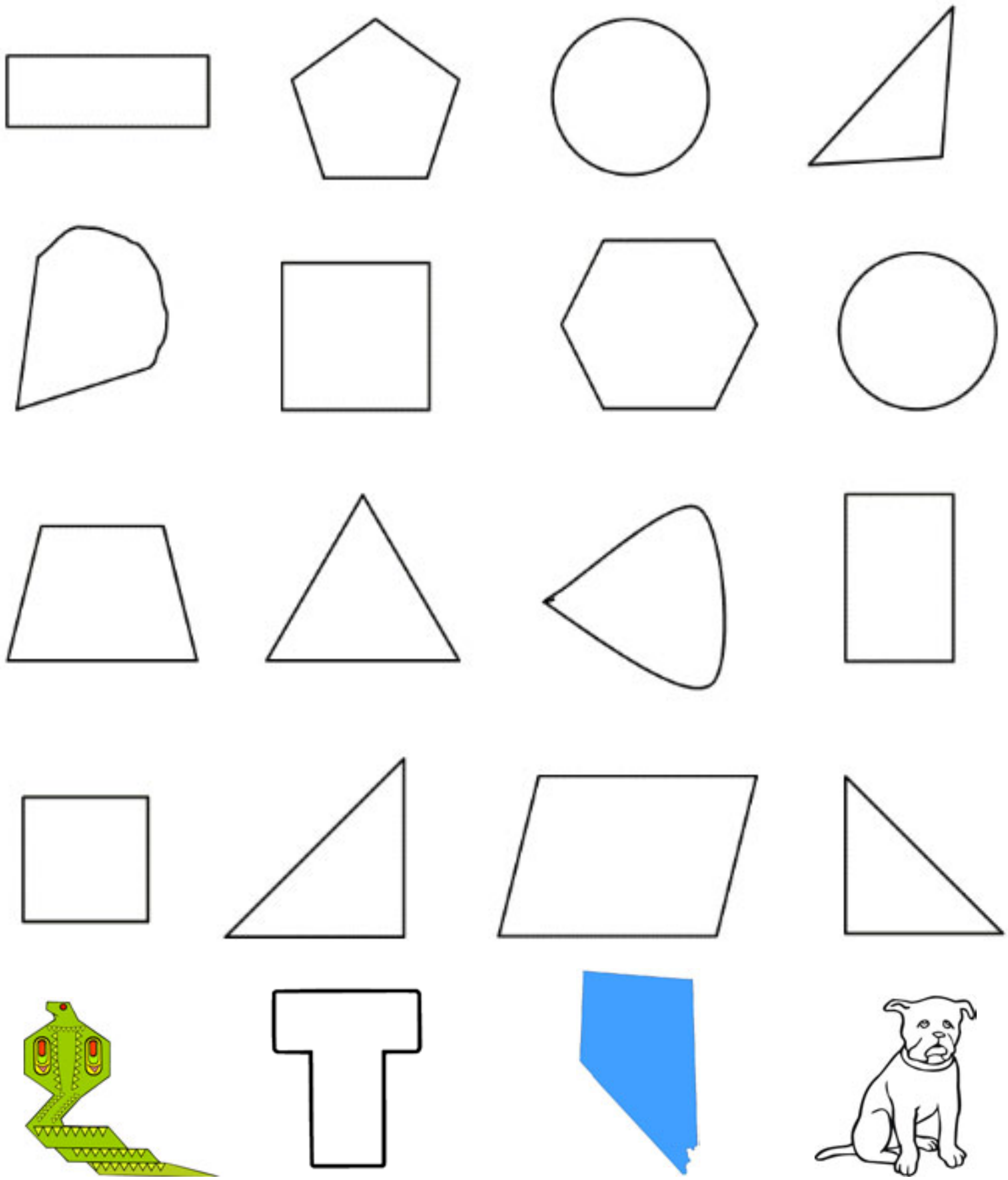
This tool is used to experiment with symmetry.

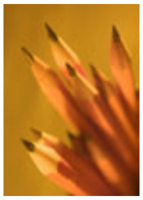
Tessellations Creator
http://illuminations.nctm.org/ActivityDetail.aspx?ID=202
This program will let you create and print out tessellations

Polygon or Not? Name _____

A polygon is a closed shape made of line segments. Cross out the shapes below that are NOT polygons.



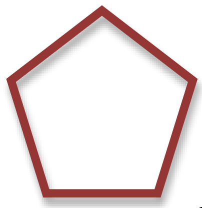
Portfolio: Closed Figures



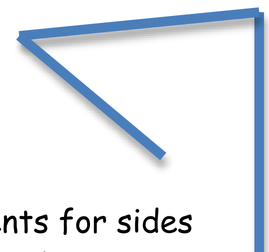
Draw a shape on your paper from line segments (no curves). Now pretend that your shape is a fenced area for your pet. Is there any place that your pet can get out? If there is, you did NOT make a closed figure. If your pet is safe, then you have drawn a closed figure.

If you did not draw a closed figure, change it into one.

Closed shapes:



Open Shape:



Plane shapes (flat shapes) made of line segments for sides are called polygons. We name them according to the number of angles and sides they contain. Where two sides meet at their ends, they form an angle. You can draw a circle around the point where two line segments meet. It will end up looking a little like a clock. An angle inside the figure is called an interior angle. An angle outside the shape is an exterior angle. The point where they meet is called a vertex.

We have special names for common polygons. You have known the names of many since you have been young.

3 sides and angles: triangle
4 sides and angles: quadrilateral
special quadrilaterals: square, rectangle, rhombus, trapezoid
5 sides and angles: pentagon
6 sides and angles: hexagon
7 sides and angles: heptagon
8 sides and angles: nonagon
1 million sides and angles: milliongon

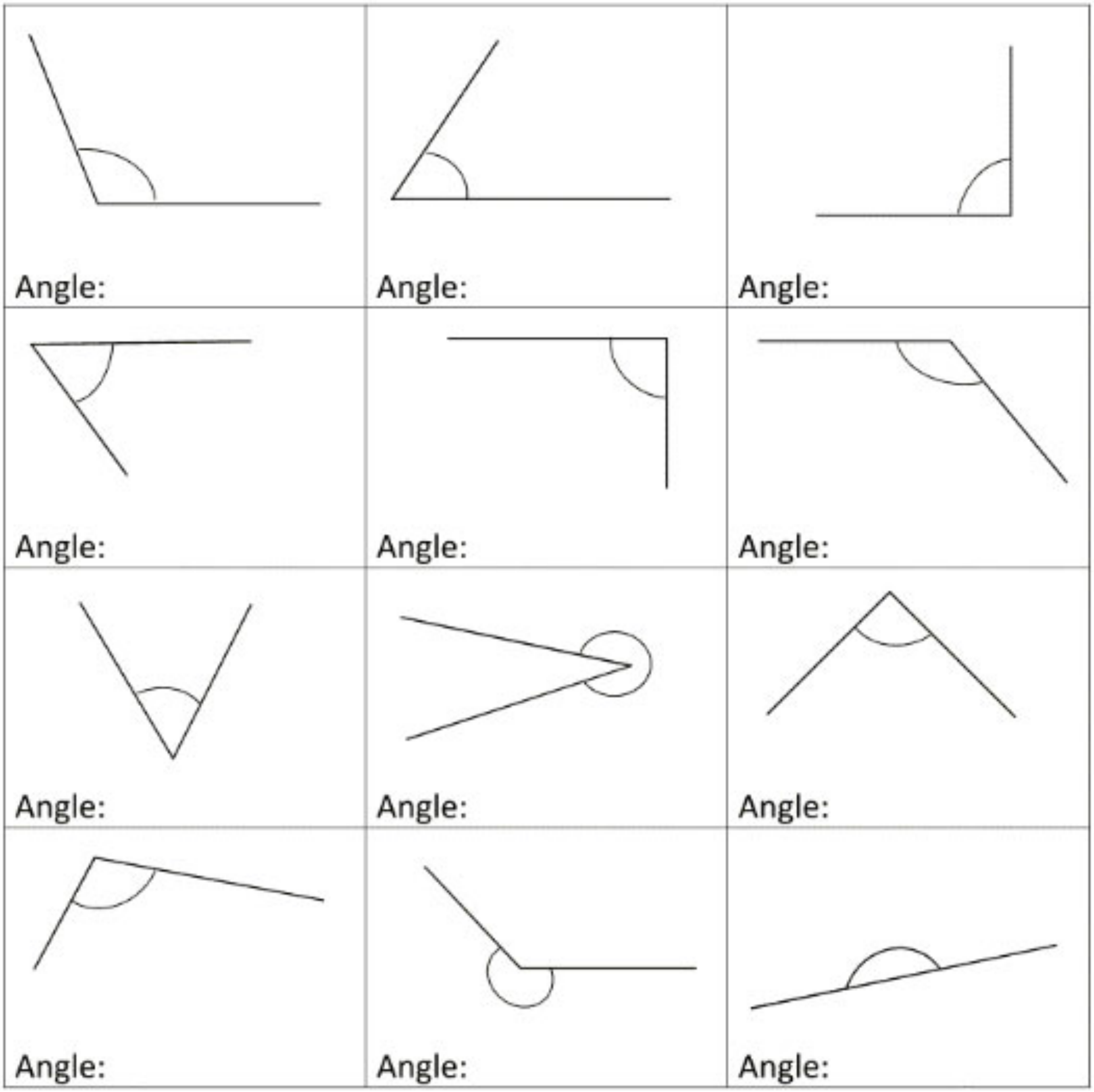
Label your personal drawing of a polygon with the following:

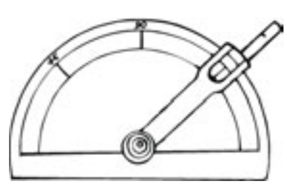
1. its name (like triangle, square)
2. one vertex (label it "vertex")
3. one side (label it "side")
4. measure one angle and label the degrees (like 54 °)

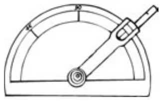
Portfolio: Angle Measurement

Name:_________

Practice using your protractor and measure these angles:



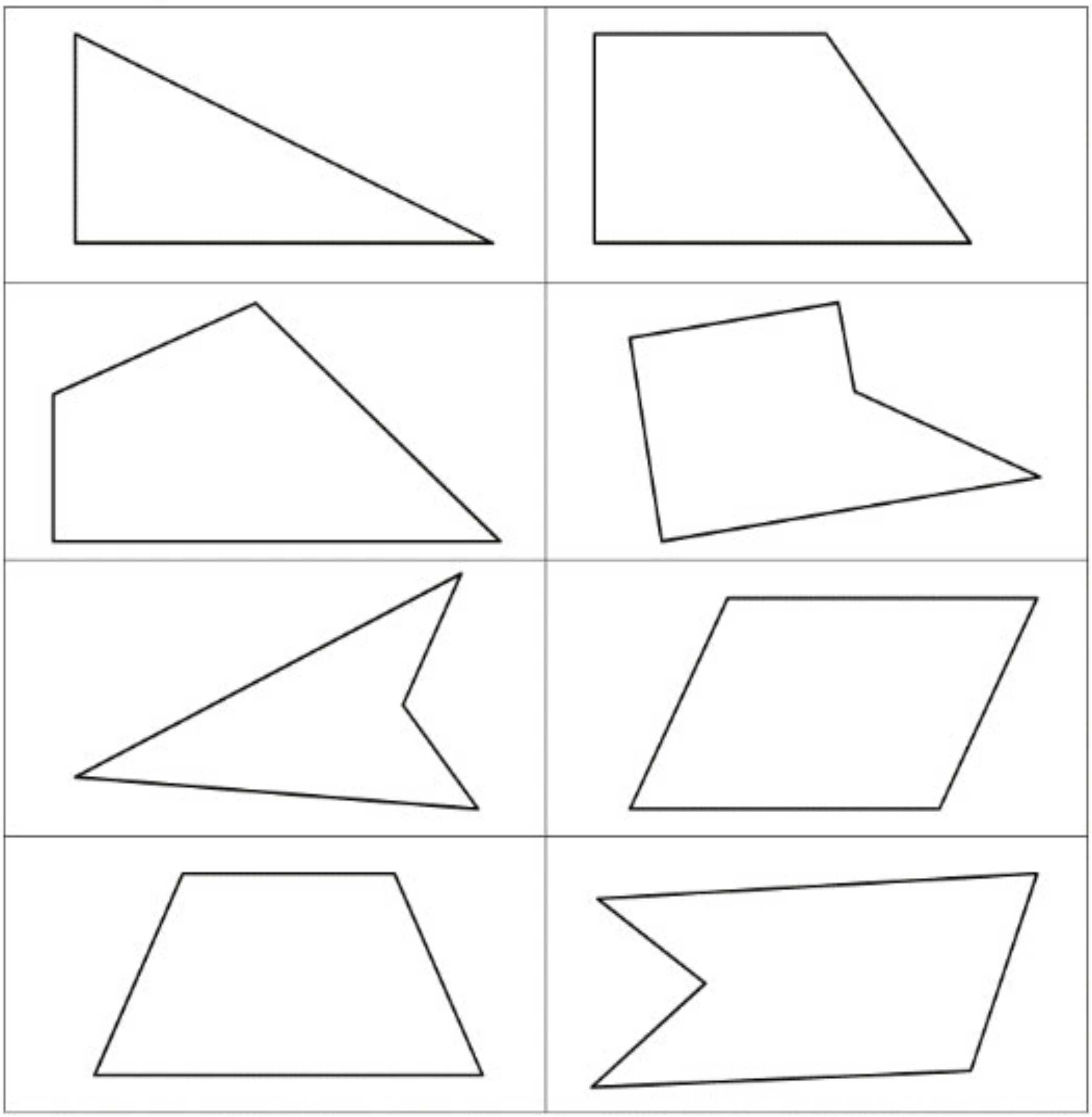




Portfolio: Interior Angles

Name:_____

Measure and label each interior angle (the ones on the inside of the figure) and add them together. In one corner write the number of sides that the figure has.

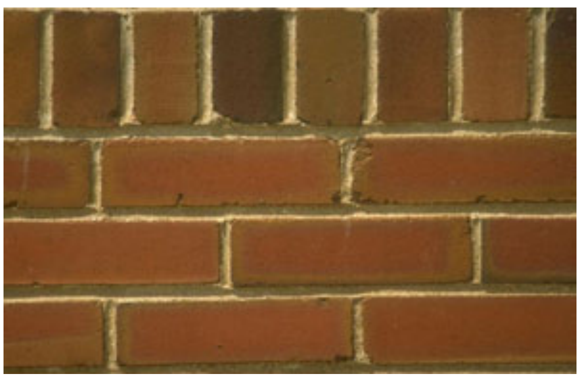


Patterns With Bricks

Make rectangular shapes (bricks) that are twice as long as they are wide. (such as 2 x 4 inches). Form groups of students. Each group should get at least 40 shapes.

The challenge is to see how many different patterns you can find that will tessellate (cover) a piece of paper. Keep each pattern for sharing.

The teacher should walk around and check on the patterns. Each group should be able to come up with at least four. Then the groups should rotate and look at others.



Polygons Make Great Tessellations

Needed: ruler, protractor, some regular polygons for this exploration. These can be paper, cardboard, or pattern blocks. There are several pages of printable polygons in this lesson. Save for **Portfolio.**

*This activity can be done in pairs, but the partners should take turns measuring and adding.

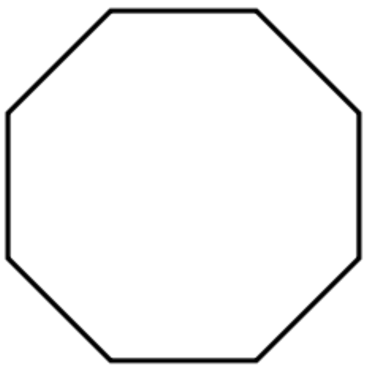
1. **Complete the following table. As you do, look for the relationships between the sum of the interior angle measures of a polygon and the number of angles in the polygon.**

Number of Angles (sketch the polygon)	**Name of Polygon**	**Sum of Interior Angle Measures**	**Measure of Each Interior Angle**
3			
4 	Quadrilateral (could also be a rhombus, trapezoid, square, or rectangle)	360 degrees	90
5			
6			
7			
8			
9			
10			
11			
12			

2. For a shape to tessellate, it must not leave any spaces or overlaps. When you cover a paper with multiples. Which regular polygons did you find that will tessellate?

3. Can you find any mathematical reasons why these shapes will tessellate? Hint: How many degrees are in a circle?

4. Will an octagon tessellate?



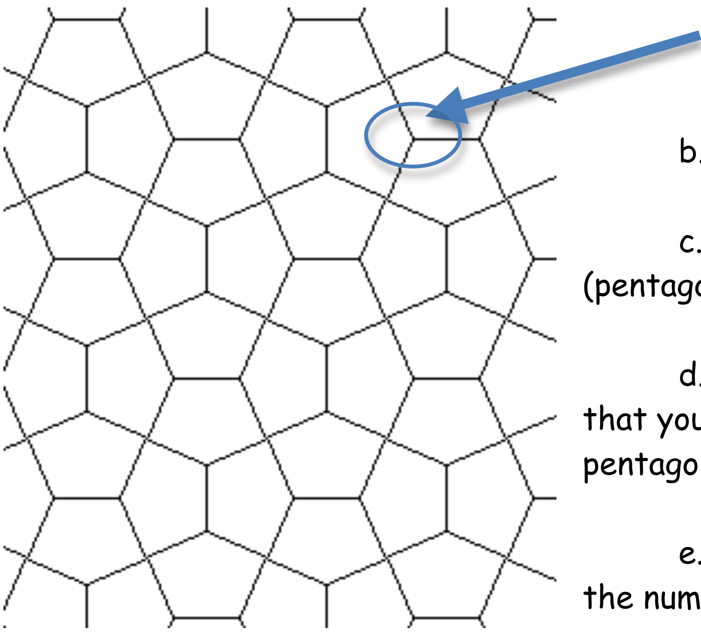
<u>Polygon Sides and Names</u>

3 Triangle
4 Quadrilateral
5 Pentagon
6 Hexagon
7 Heptagon
8 Octagon
9 Nonagon
10 Decagon
11 Undecagon or Hendecagon
12 Dodecagon

Round and Round the Vertex: Naming a Pattern

The previous work explored regular tessellations. Regular tessellations cover a plane using only 1 type of regular polygon. Tessellations can also be done using 2 or more types of regular polygons. The arrangement of polygons around every vertex point must be identical in both semi-regular and regular tessellations.

A vertex point is where all the polygons meet. Go around a vertex point clockwise and name the polygons as you go. Look at the following example:



a. Circle a vertex.

b. Go around it clockwise.

c. What are the regular polygons? (pentagons)

d. As you go around, you should discover that you started in a pentagon, then another pentagon, then a third pentagon.

e. To put this in correct terms, count the number of sides of the pentagon. (5)

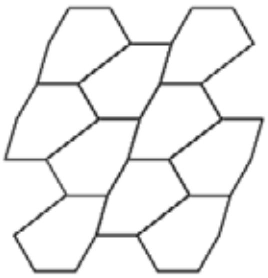
The way this is written is (5,5,5) which stands for (pentagon, pentagon, pentagon)

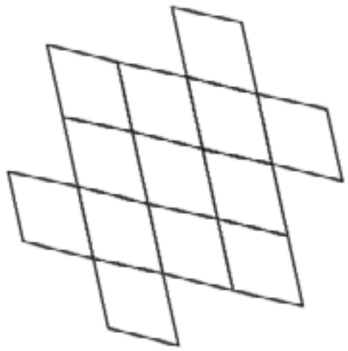
Try the examples on the next page.

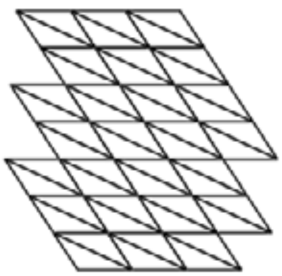
Portfolio:

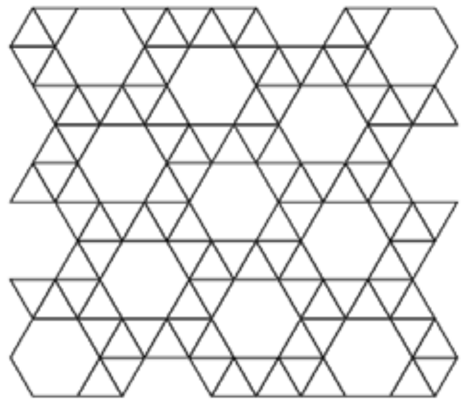
Patterns of Polygons   Name: _______

Circle a vertex and label the pattern.





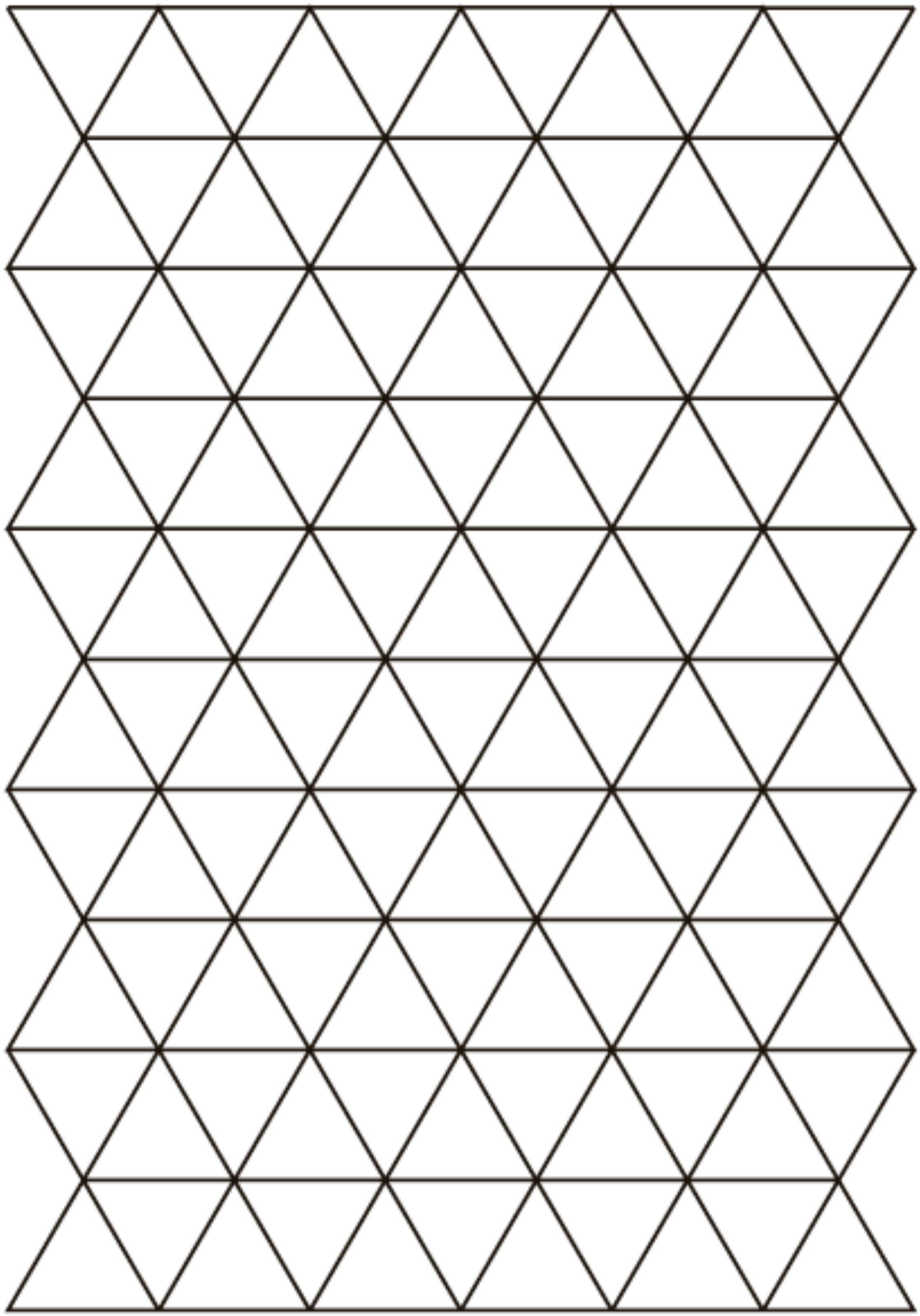


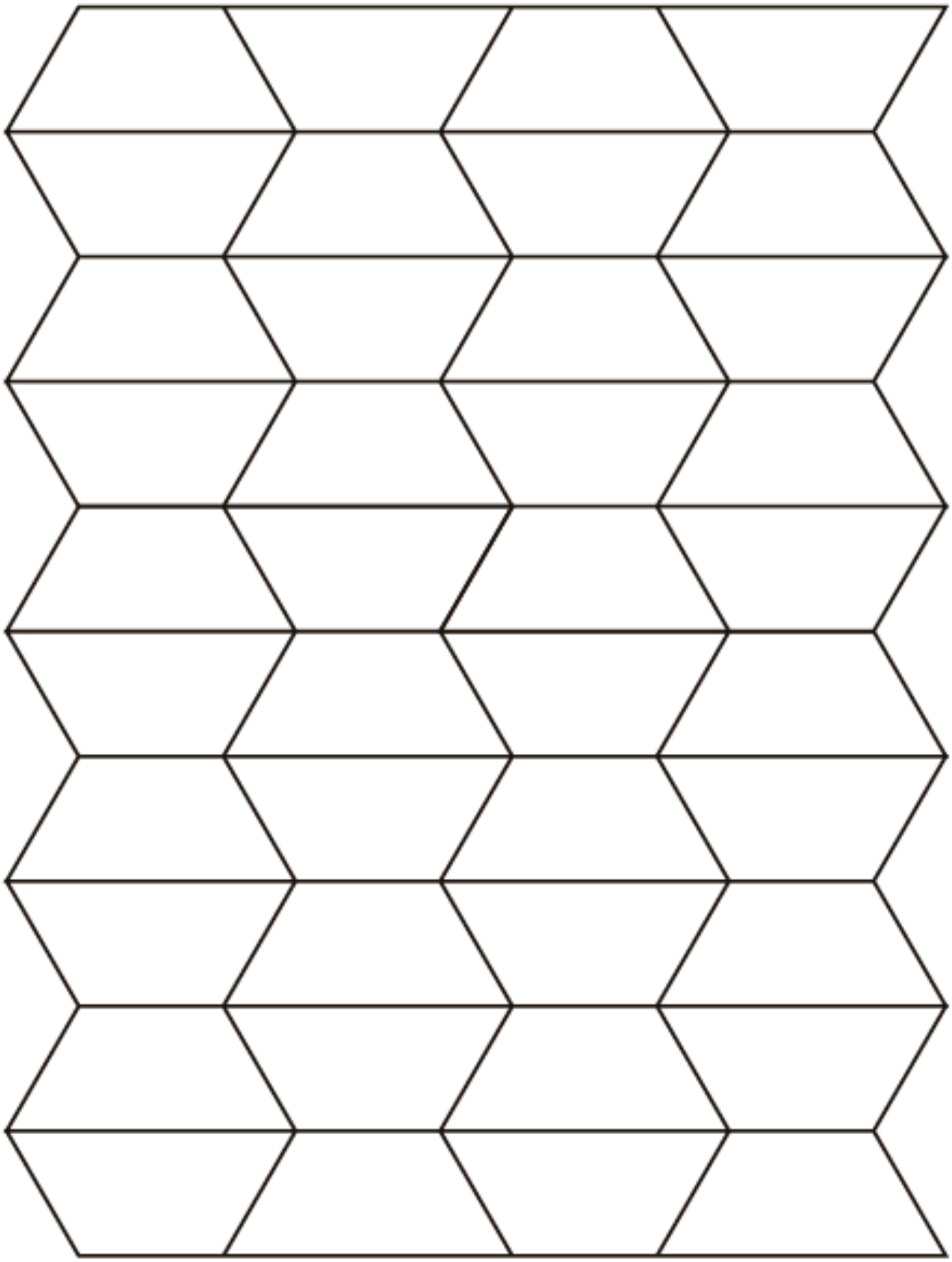


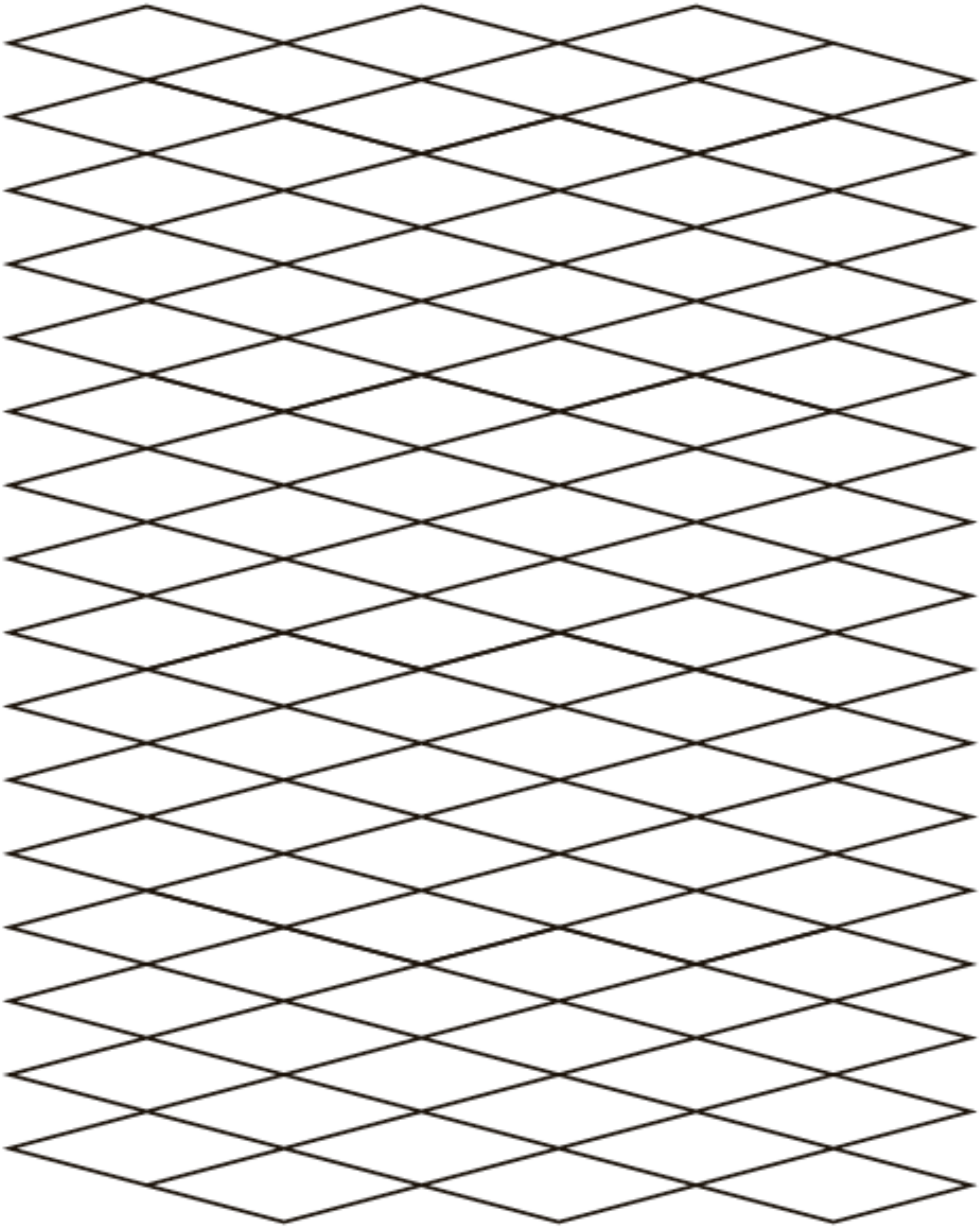
Now, sketch the following patterns:

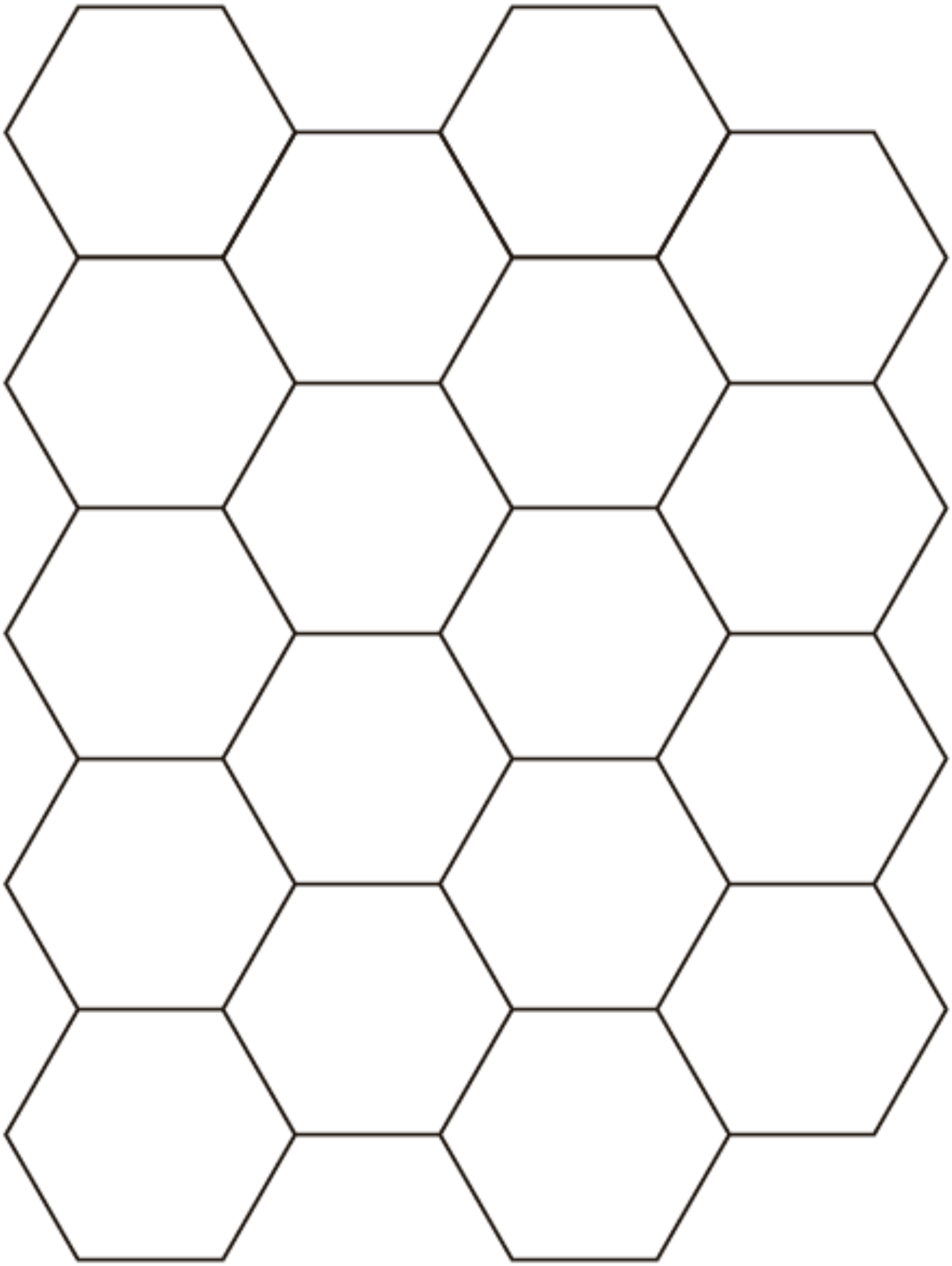
1. **4,4,4,4**
2. **3,3,3,3,6**
3. **3,3,3,4,4**

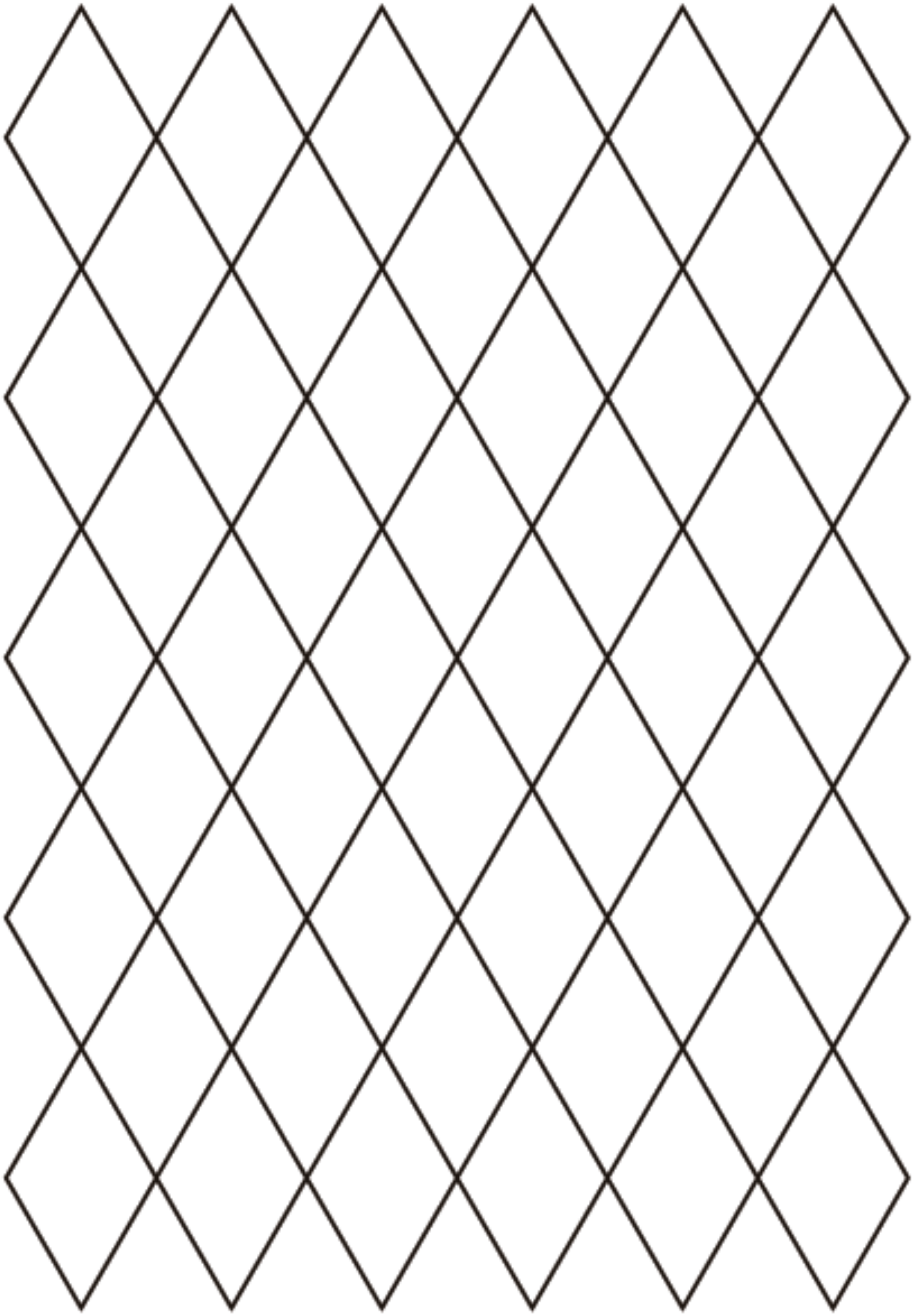
PAGES of PATTERNS

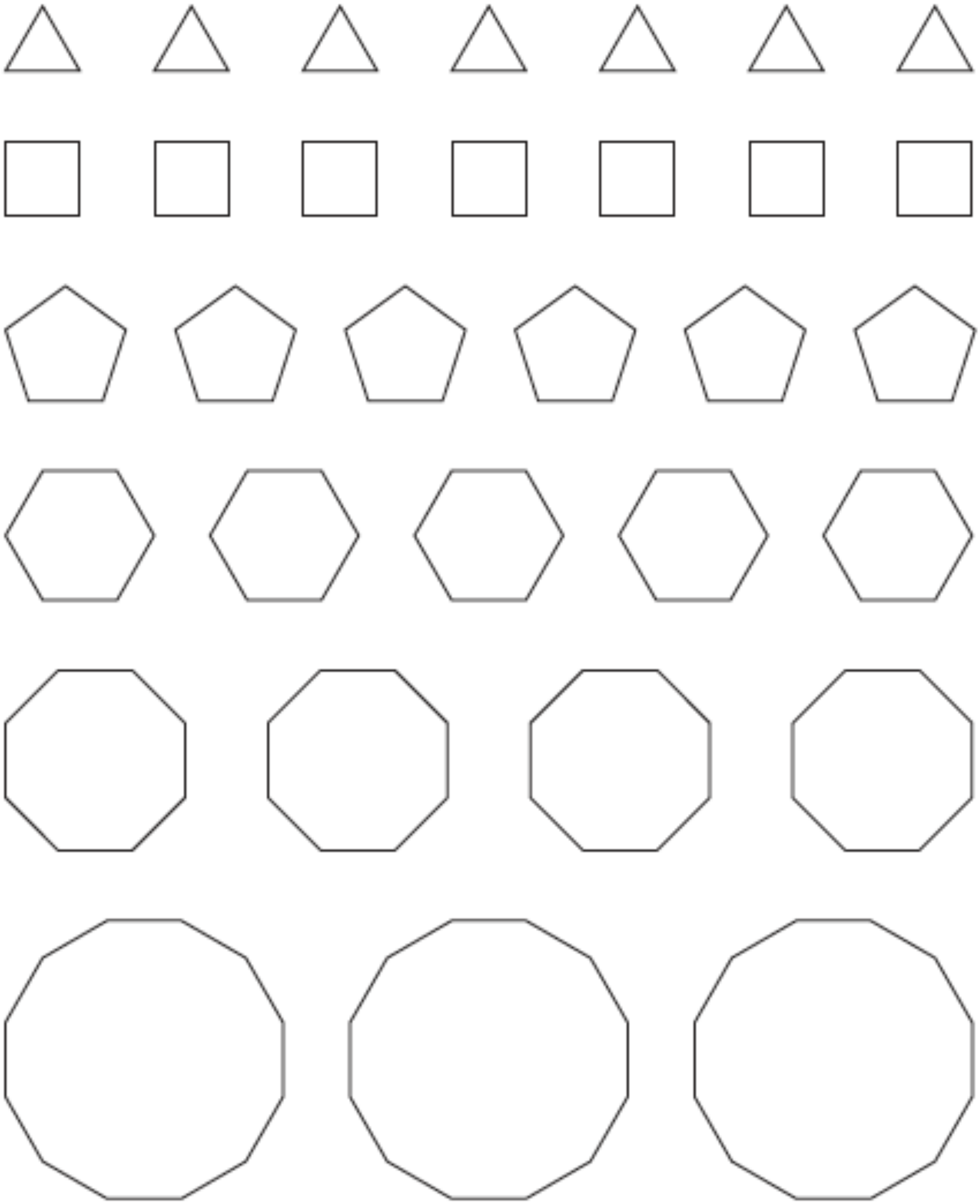




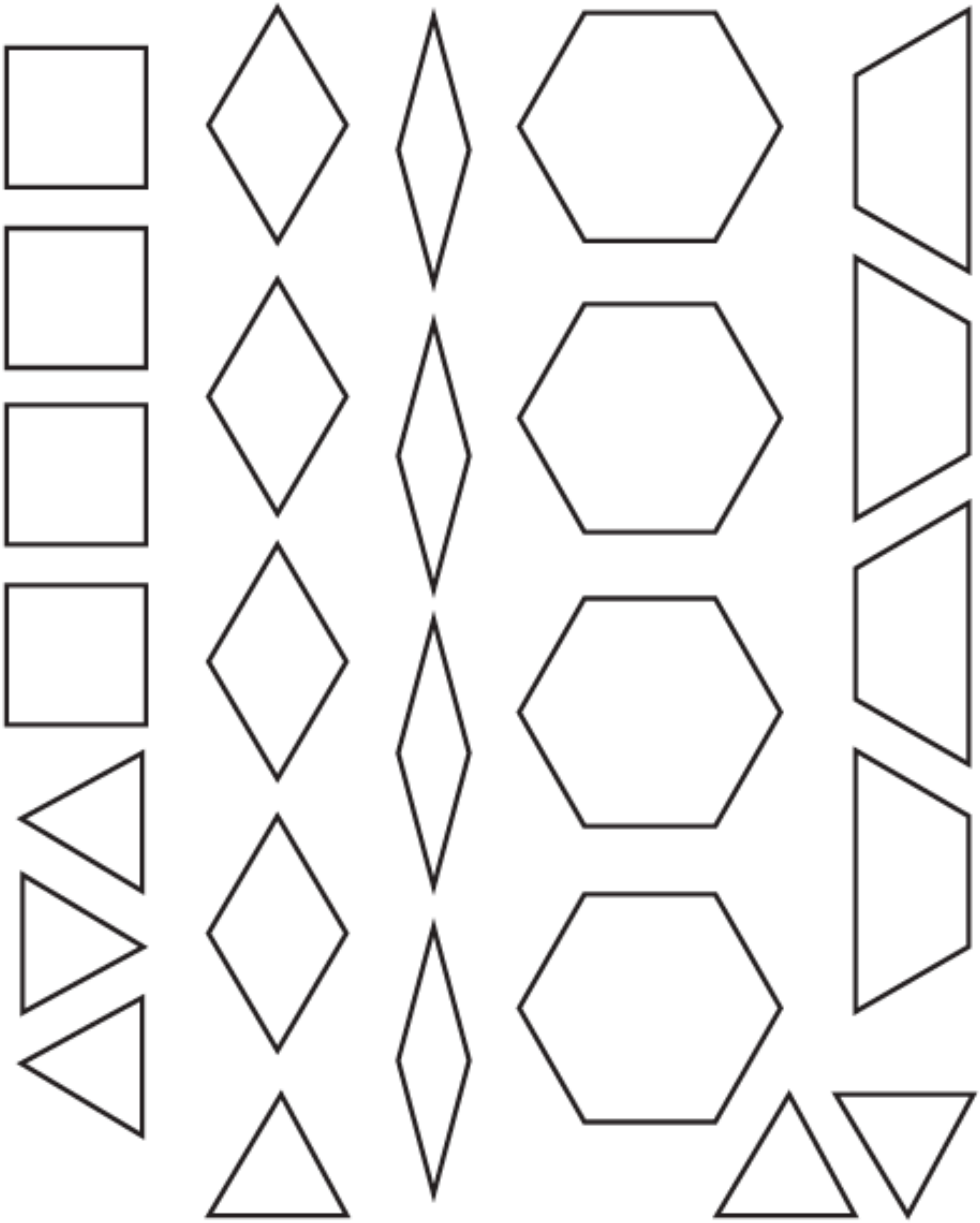








Pattern Blocks



What IS a Tessellation?

Patterns of geometric design surround us. They are in our flooring, our quilts, our walls. Some are very simple; others intricate and intriguing. In this unit we will explore a special kind of geometric pattern called tessellations.

A tessellation covers a plane surface with a repeated shape. The shape must not leave gaps between the tiles and must not overlap.

Of course, this looks easy if you are using squares or rectangles. But there are many other shapes that make extremely complicated and pleasing patterns.

The word tessellation comes from the Latin language. A tessalla is a small square tile or stone that was used in ancient Roman mosaics. While three-dimensional shapes will tessellate, we are going to focus on the two-dimensional shapes, which form a plane tessellation.

This unit is intended to be visual and hands-on. Please take the time to provide the materials needed.

Begin this lesson with the PowerPoint:

"Intro to Tessellations and Definitions"

Shapes and Tilings

As you work with the polygons, show students on a pattern block where a vertex is. Younger students will think of it as a corner or point of a shape; older students should be led to the geometric concept of the point where two of the line segment sides meet to form an angle.

When you continue to use the terms, students will pick them up almost effortlessly.

Review with students some basic geometric shapes:
Triangle, square, rectangle, trapezoid, pentagon, hexagon

Concepts: polygon, vertex, regular polygon, plane, tiling

Definition: ***What is a Tessellation?***

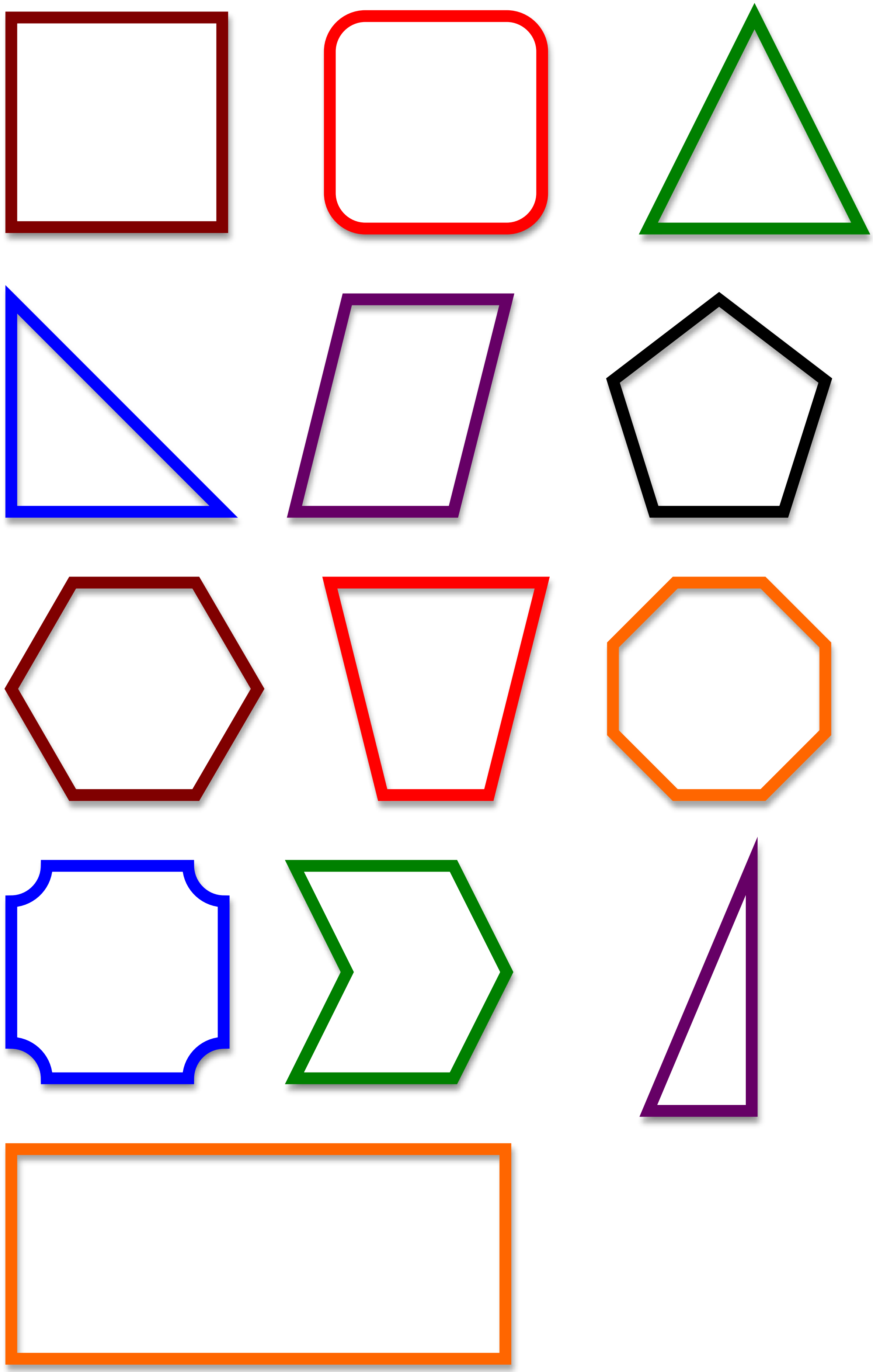
A tessellation covers a plane surface with a repeated shape. The shape must not leave gaps between the tiles and must not overlap.

Divide the class into groups.
Each group should receive a set of three different polygon shapes and paper. There should be at least 6 of each shape. These should be identical. Pattern blocks may also be used.

Group members should lay out these shapes and discover which ones will tessellate a plane. Only one shape should be used on each paper.

Share the results with other class members.

There are templates for copying and making paper pattern blocks in this unit. To make these paper blocks more useful, make each polygon type a different color.

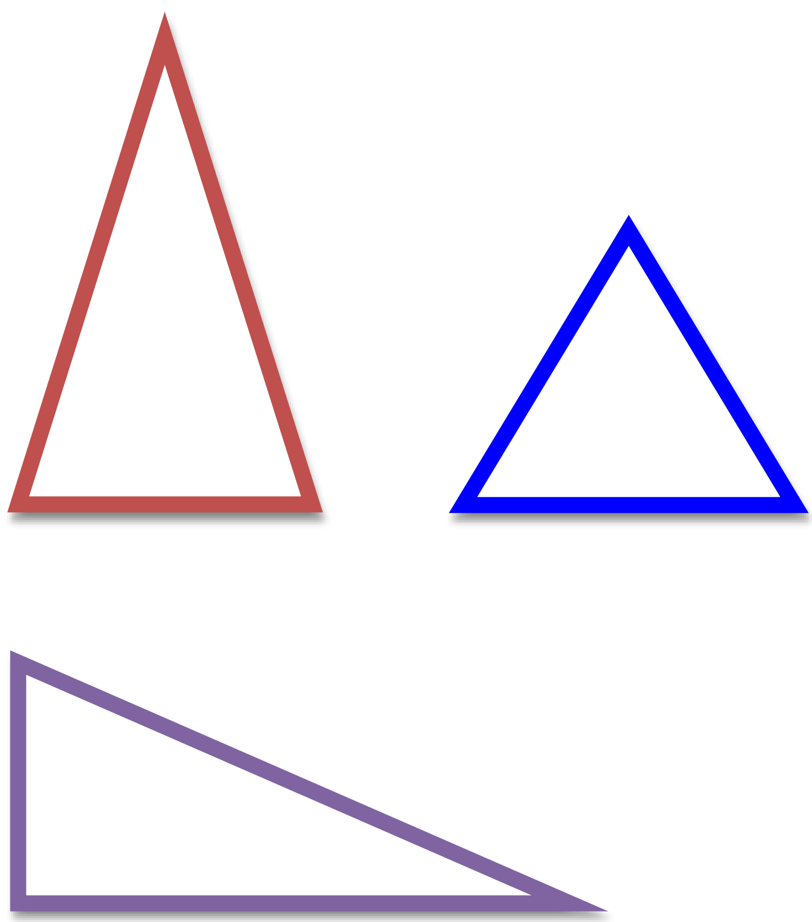


Why will certain shapes tessellate while others will not?

Draw four different kinds of triangles on index cards: an equilateral triangle, a scalene triangle, a right triangle, and an obtuse triangle. If you do not know the types of triangles, you may use the following page of examples. Upper grades may want to use what they know about constructing triangles with compasses, protractors, and rulers.

Label the triangles equilateral, scalene, right. Cut out a pattern of each one. Using ONLY one type triangle, try to tessellate it and cover a paper. Then try the others in turn. Use only one kind of triangle per paper.

Do all the triangles tessellate?

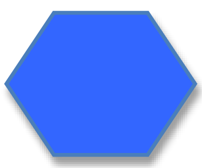


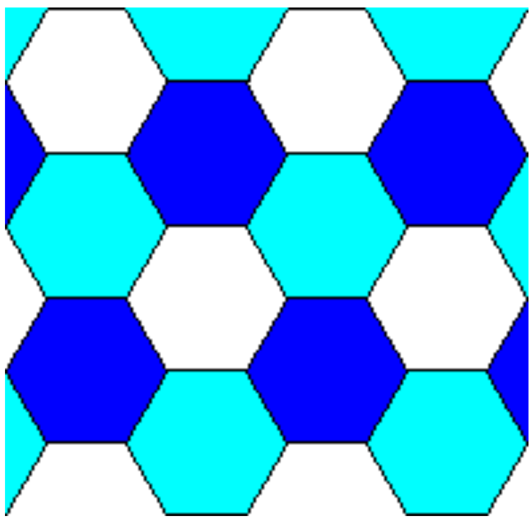
Tessellations

If a shape **tessellates**, it means that it will fit together with other identical shapes to fill up any two-dimensional shape leaving no gaps. The shapes may not overlap, although they may flip and move. Sometimes it is called **tiling.**

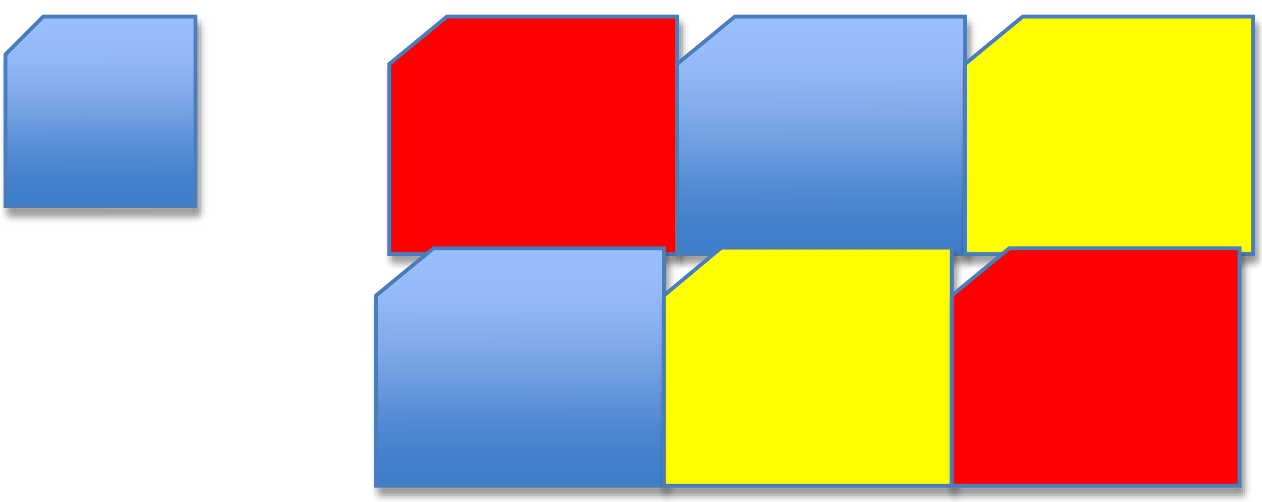
There are many shapes that do NOT tessellate and there are many different shapes that DO.

These hexagons tessellate:





This modified square does NOT tessellate. There is a missing piece.

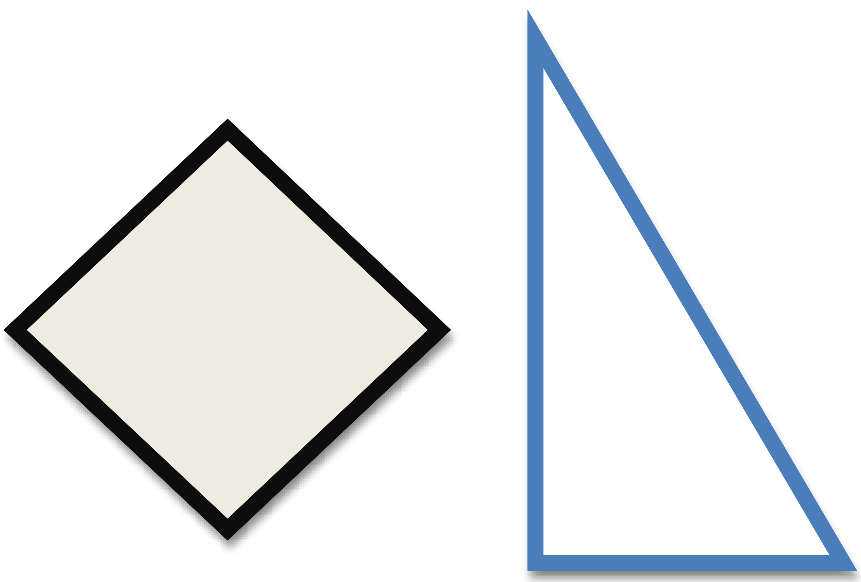


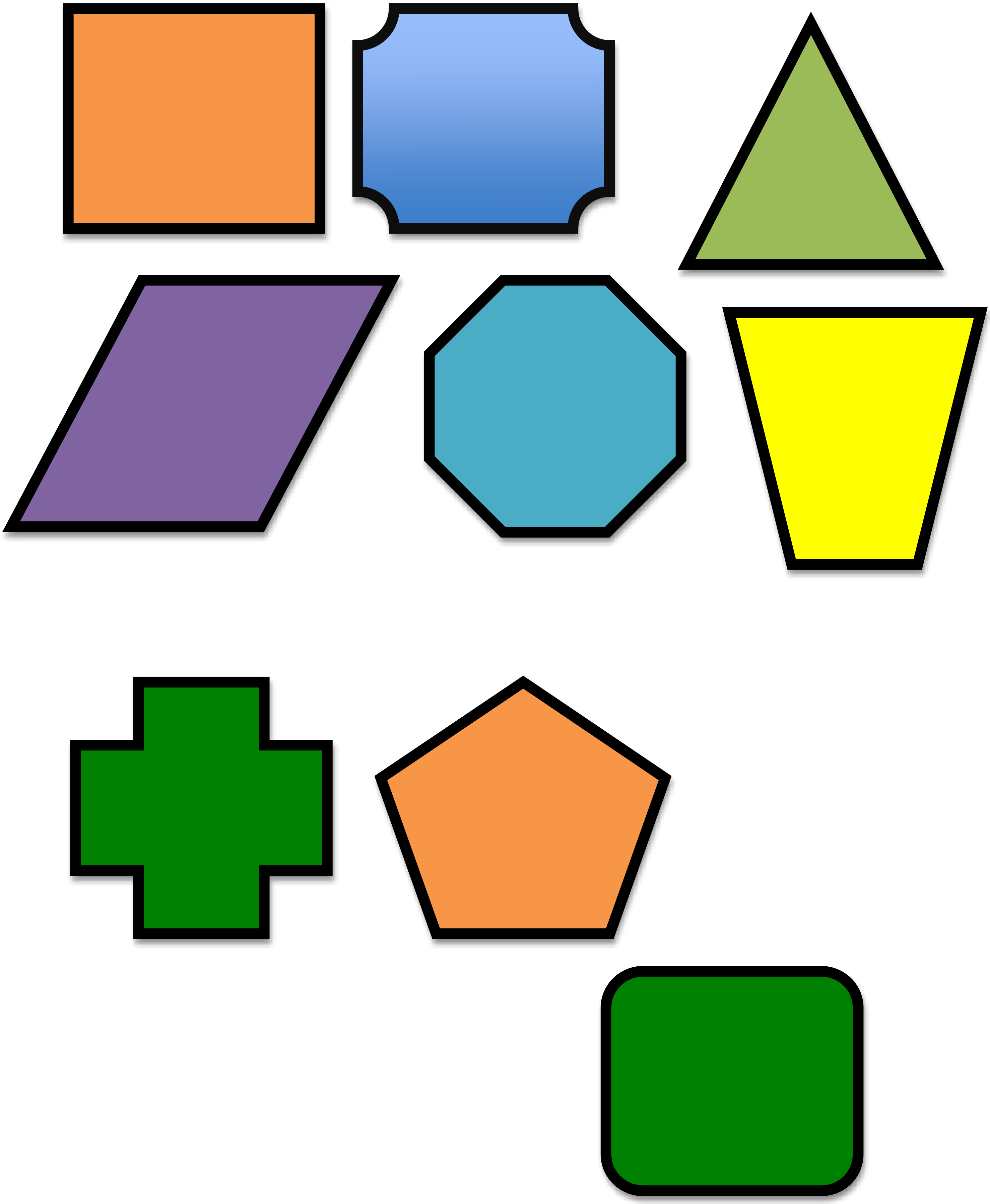
TESSELLATIONS and POLYGONS

Directions: Divide the class into small groups.

Give each group a set of four polygons to cut out.
You can also use pattern blocks, although most of them will tessellate. Have students make templates on cardboard of each polygon. (Cereal boxes work fine.)

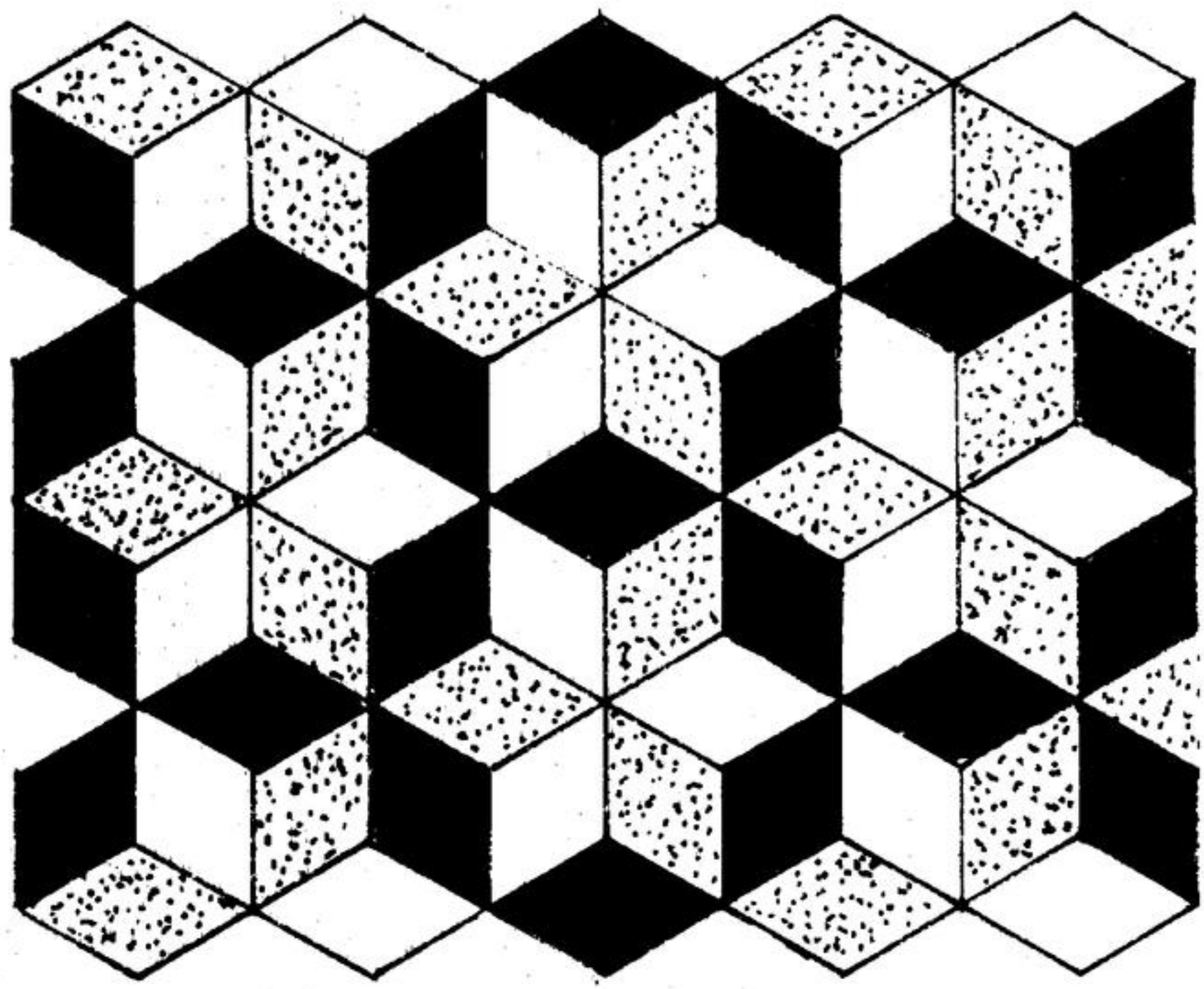
Now try to tile a plane surface by outlining each polygon repeatedly. Which will cover a plane? Share the results with the class.





Portfolio: Color Makes a Difference

Colors can make a big difference in how a tessellation is viewed.

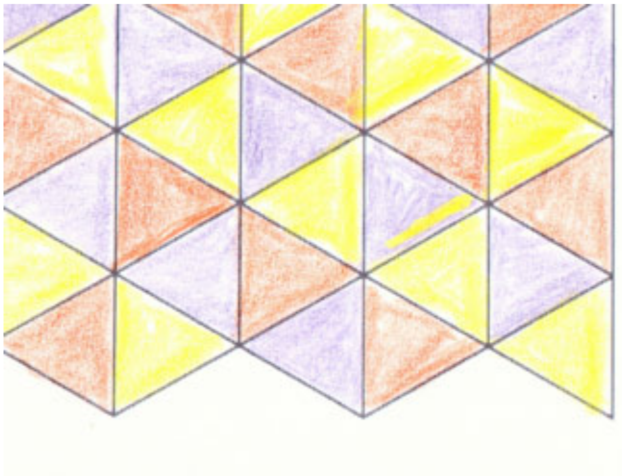


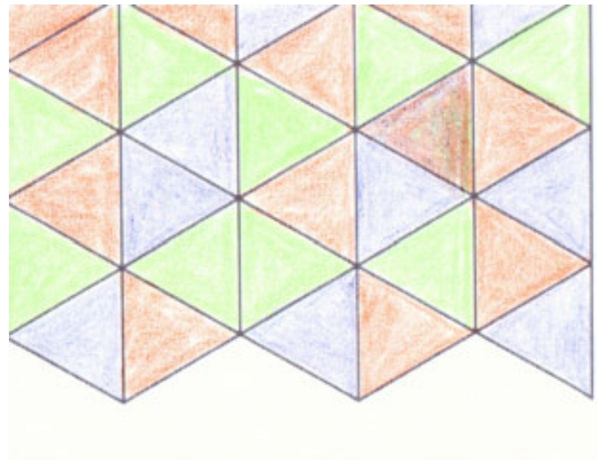
Directions:

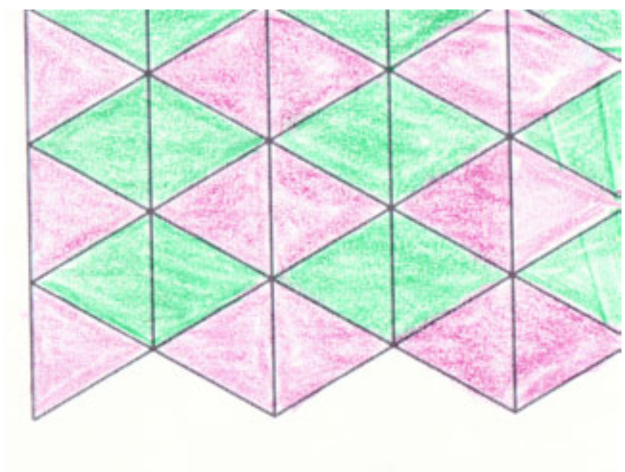
There are three sets of rhombus (diamond) tessellations on the next page. Use at least two colors on the tessellations to make each one look different. Try using different colors.

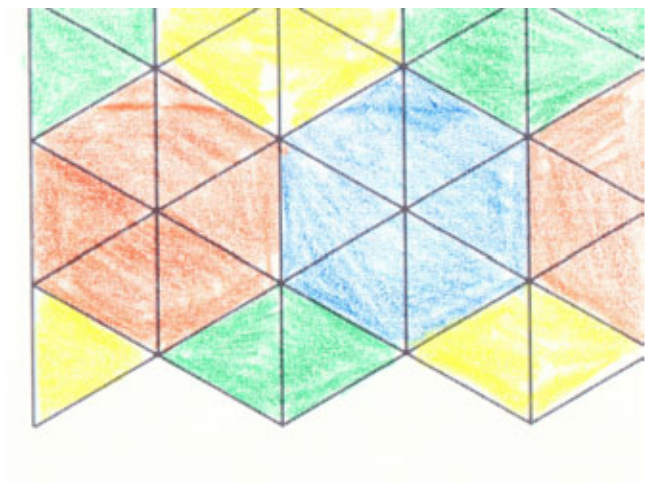
Here are some small samples done by students to show what a difference color patterns can make in a finished result. These are all the same tessellation, but students put on color that make them all look different.

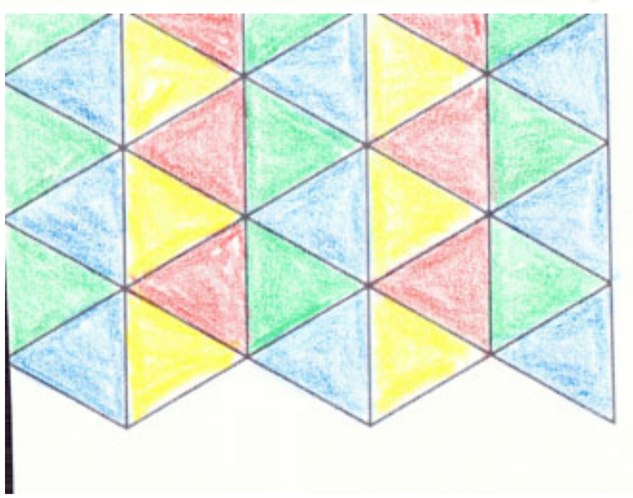
Add this paper to your portfolio.

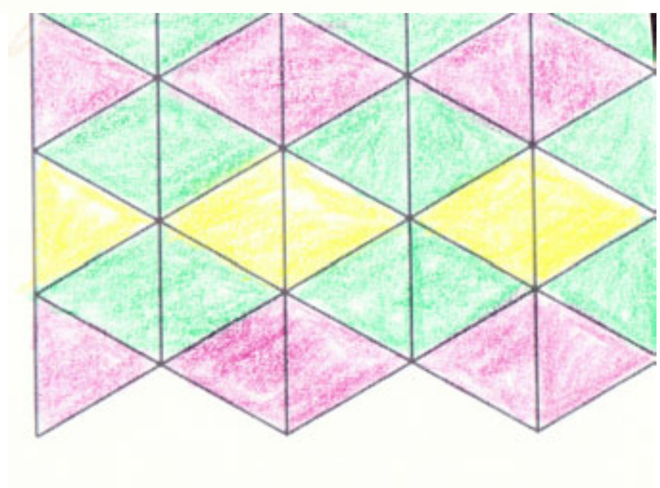








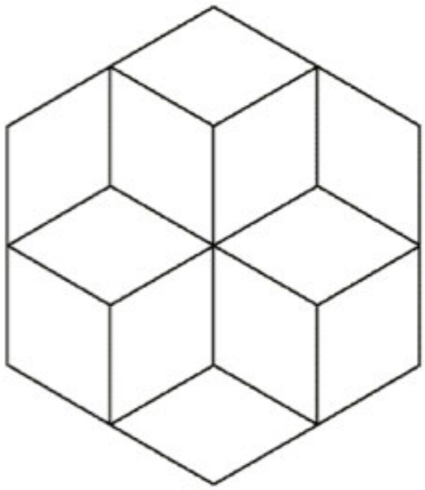


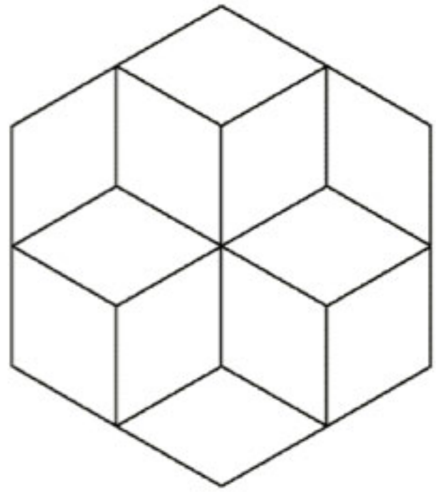


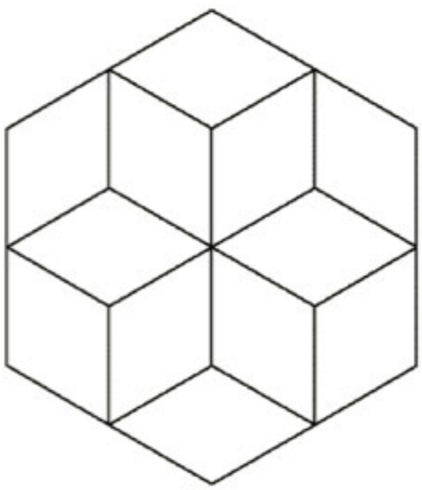
Option: If you would like to work with bigger shapes, copy the page after the next and give six to each student.

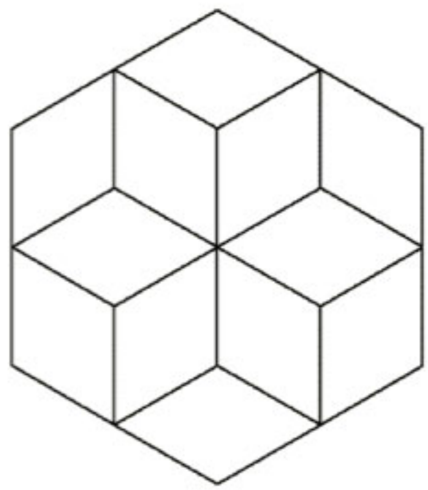
Color Makes a Difference WS

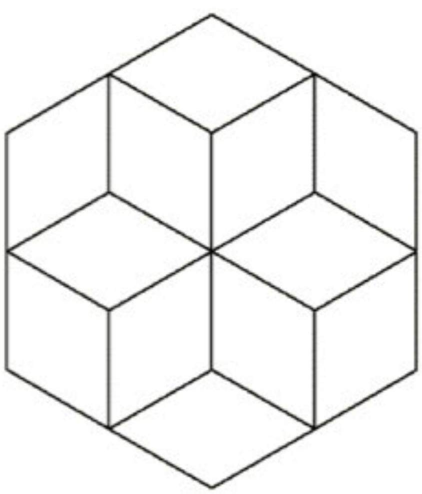
Name: _____________

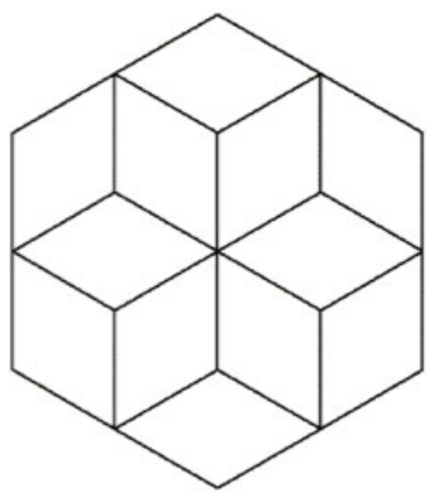




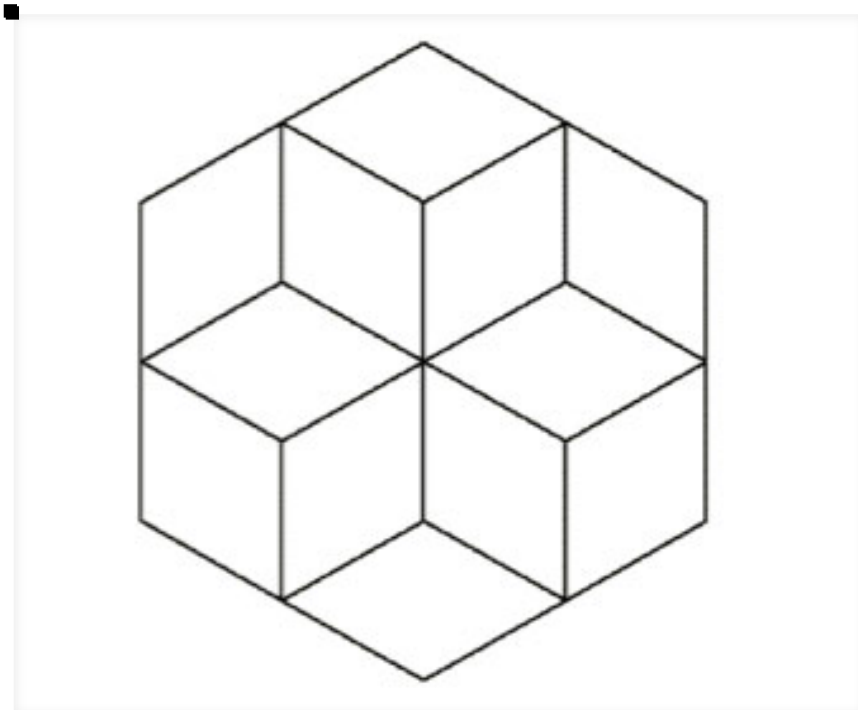


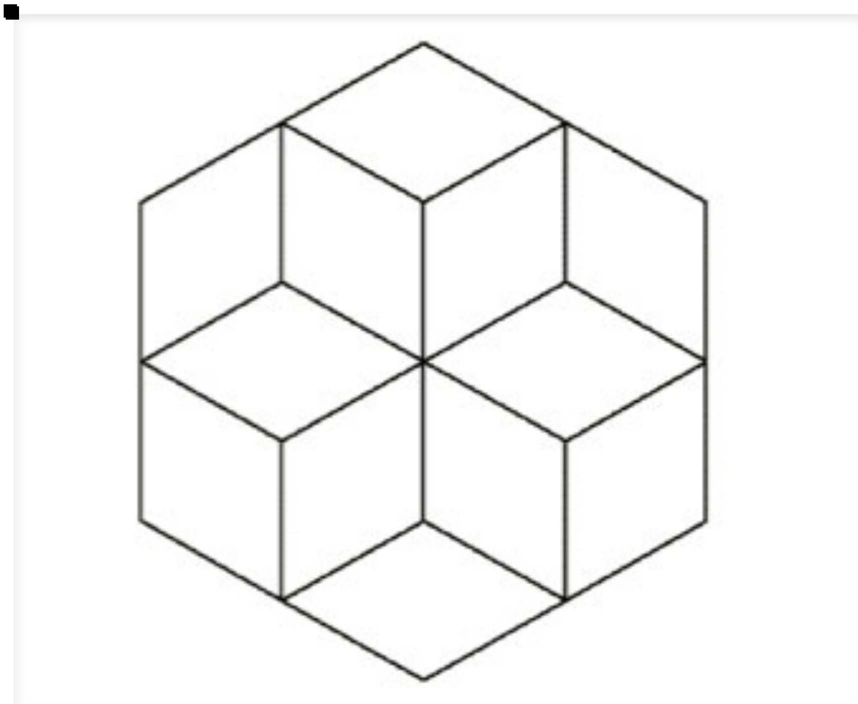






Color Makes a Difference Name: ___________________





TYPES OF SYMMETRY

Symmetry is easy to illustrate, but hard to define in words that make sense. So look carefully at the illustrations. **Show the PowerPoint "Symmetry and Art."**

Symmetry is defined as "the correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point; regularity of form or arrangement in terms of like, reciprocal, or corresponding parts." Symmetrical patterns occur in nature and are also used by artists, musicians, mathematicians, and craftspeople.

Plane Symmetry:
This involves moving all points around on the plane so that their positions relative to each other remain the same. You can illustrate this concept by placing your hand flat on a table. Now move it to the right. You moved all the points in your hand at once, keeping the points the same relative distance from each other. So all your fingers came along, and your thumb is still in the same relationship to your little fingers.

Symmetries preserve distances, angles, sizes, and shapes.
If there is at least one symmetry (reflection, translation, rotation) that leaves the pattern unchanged the object, etc., is symmetrical.

Translations & Slides:
If an object is translated it is merely moved without rotating it or reflecting it. Every translation has a distance and direction.

Before translation K ***After translation*** K

Before translation K

After translation K

HANDS ON:

Supplies needed: small piece of cardboard or paper, scissors, graph paper

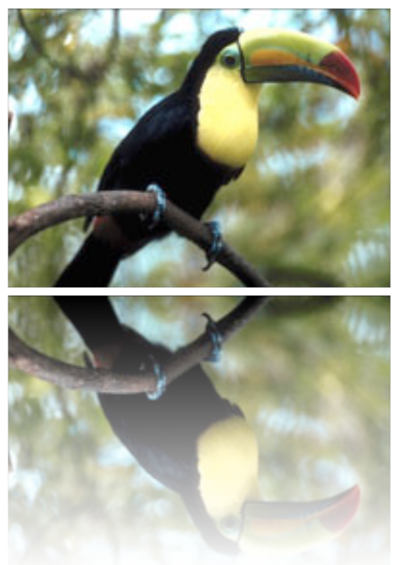
Directions: Label the four sides of the graph paper with the words up, down, left, right. Cut a small object out of the cardboard. Place it on the graph paper and trace it. Now move it over in a straight line, and then up or down. Do NOT rotate or turn it. Record your directions for moving in pencil on the graph paper. You may use the word "unit". (For instance, you moved your object 5 units right and 9 units down). Trade your paper with other students and see if they can figure out how you moved it.

Reflective Symmetry:

We all know reflections from the mirrors in our lives. To reflect an object means to produce its mirror image. Every reflection has a mirror line. If an object is far from the reflection line, the distance between the object and its mirror image will be greater than if it is close to the reflection line.

When we stand up straight, our bodies have a line of symmetry dropping vertically from the middle of the top of our heads. That means if we were folded in half, the two halves would match. Many objects have one, two, or even three lines of symmetry.

Before reflection

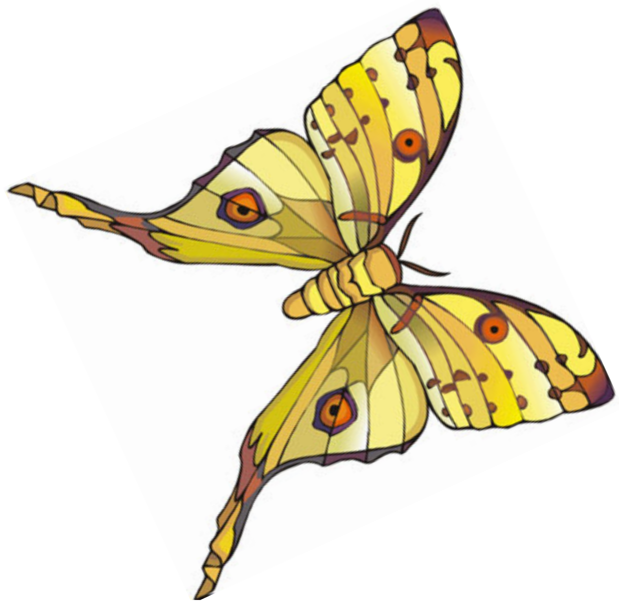


After reflection

HANDS ON:
One way to make a reflection is to fold a piece of paper and draw a shape on one side. Now trace it on the other side of the fold.

Butterfly Crafts.
Supplies needed: construction paper, pencils, paint
Directions: Fold a piece of construction paper in half. Write your name in large letters along the fold. (These look better in cursive, but manuscript will work.)
Now paint the letters with a lot of paint. Small squeeze bottles with tempera paint works great. Work quickly. While the paint is still wet, fold the paper over the paint. Lightly rub it. Now open it. There should be a great reflection of your name.

Put the lines of symmetry on these butterflies. Look for examples of symmetry in nature.



What about each honeycomb cell? Or each plate on a turtle's shell?

Name: ____________________

Activity: Alphabet Reflective Symmetry

Many of our letters have lines of symmetry, some more than one. But that can depend on what font you use. Put lines of symmetry on the letters below. Not all of them have lines of symmetry. Some have more than one. Using a small mirror can help.

A B C D E

F G H I J K

L M N O P

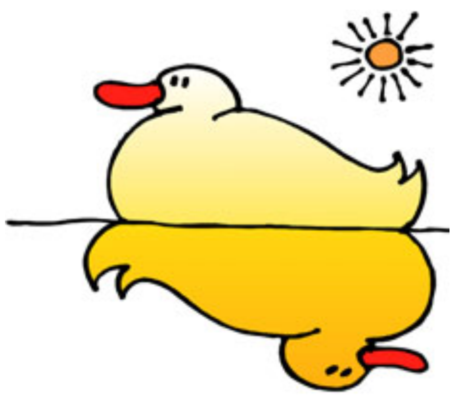
Q R S T U

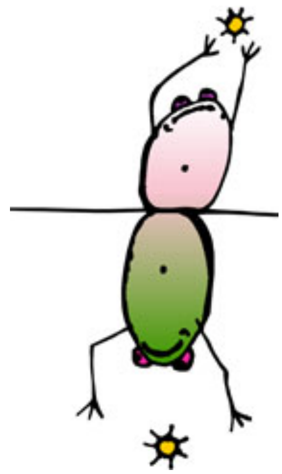
V W X Y Z

Please note that in different fonts, some of the letters such as C and V do NOT have lines of symmetry. Ask students to redraw some of the letters that they think could have symmetry in another style or font.

Further exploration: What about the lower case letters?

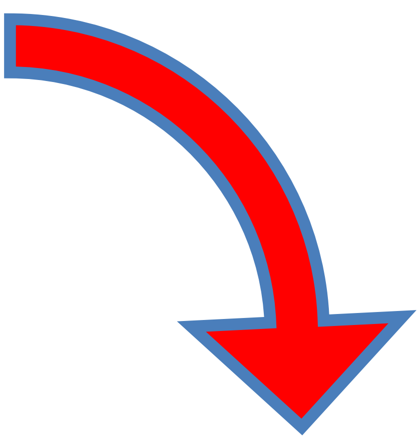
Check out these fun examples. At first, they appear to have reflective symmetry. But if you look closely, you will see they do not.





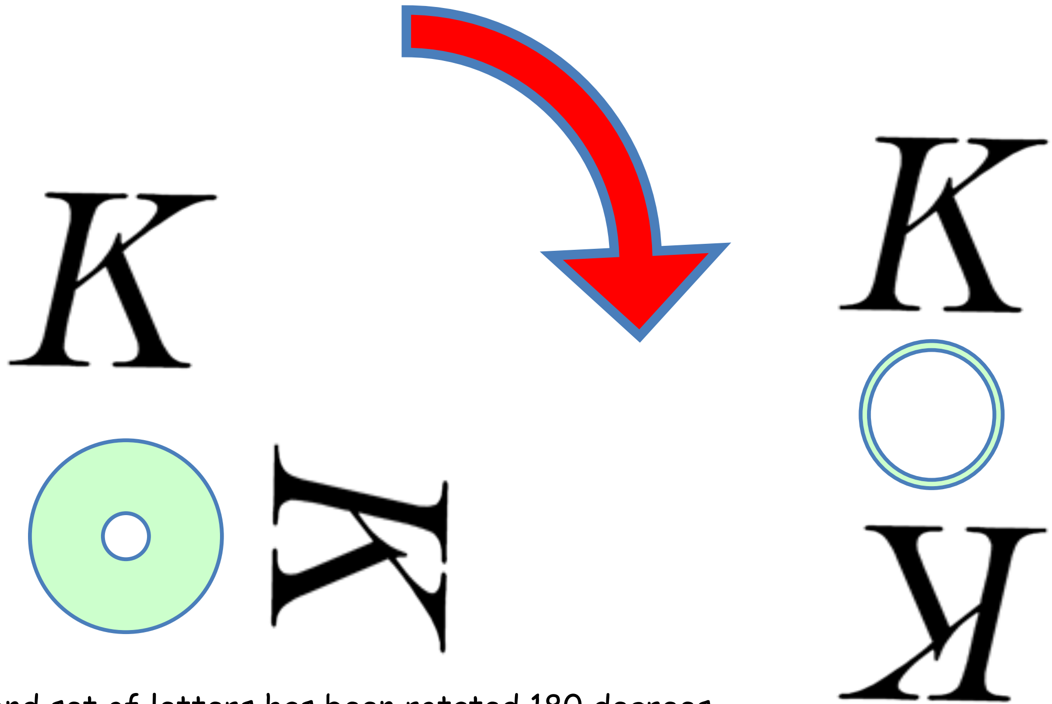
Put a small mirror along the line drawn on the ducks and frogs above. Now can you see the differences?

Reflections can also be called **flips**.



Rotational Symmetry

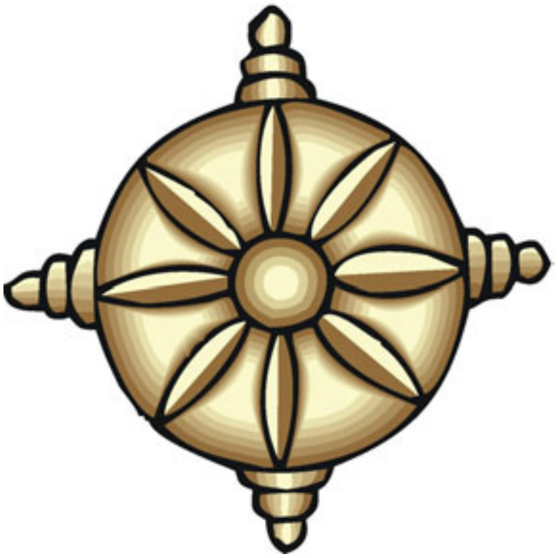
Every rotation has an angle and a center. To rotate an object means to turn it around that center. This first set of the letter K has been rotated clockwise 90 degrees.



The second set of letters has been rotated 180 degrees.

Rotations can be used to make beautiful and complex patterns. However, to make them create a circular pattern that does not overlap, the number of rotations must add up to a total of 360 degrees.

This design uses both reflective symmetry and rotational symmetry.

The two compasses above show rotational symmetry. Notice on the first compass there are two sets of triangles that have been rotated.

At first glance the patterns on the left look like they have reflectional symmetry. But they do not. The blue squares show a slide reflection on the plane of all four.

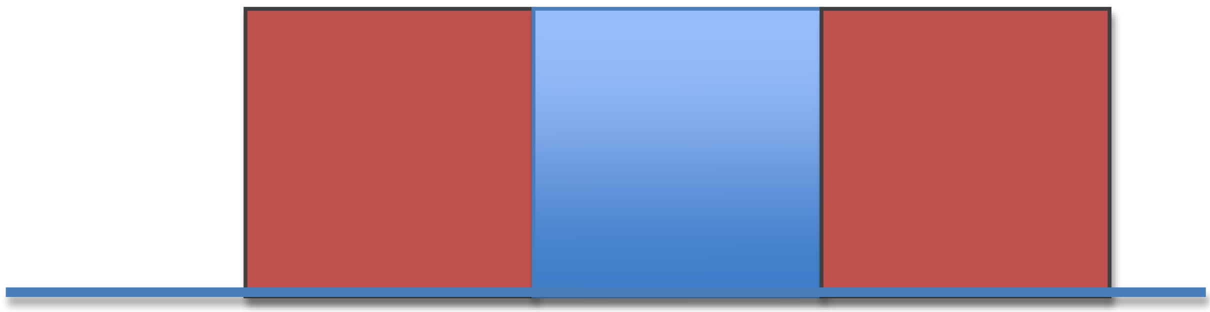
Look inside each of the four square areas. Each of the squares shows rotational symmetry around its center.

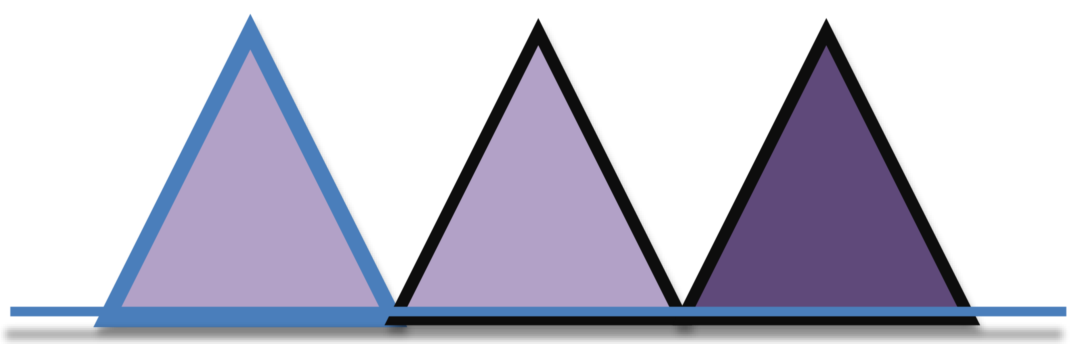
Translations and Slides:

A **slide** takes place when the shape is moved up, down, left, or right along a line. It should not overlap. This is also called a **translation.**

Directions: On a piece of paper, draw a line.

Choose a shape and "slide" it along this line. Do not overlap. Does your shape tessellate along a line?





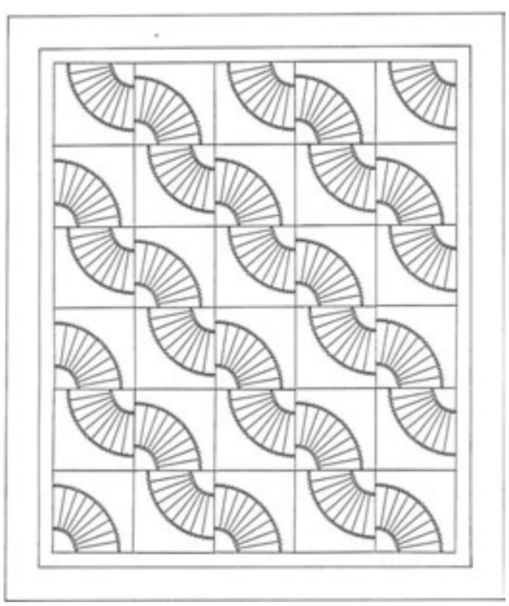
(Squares and rectangles will, most others will not.)

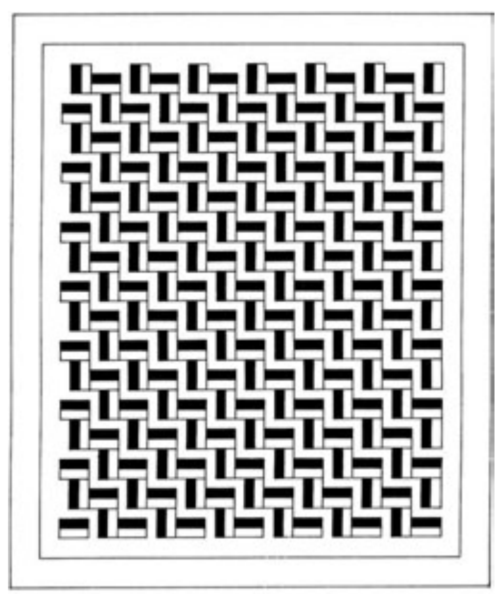
Reflections and Flips:

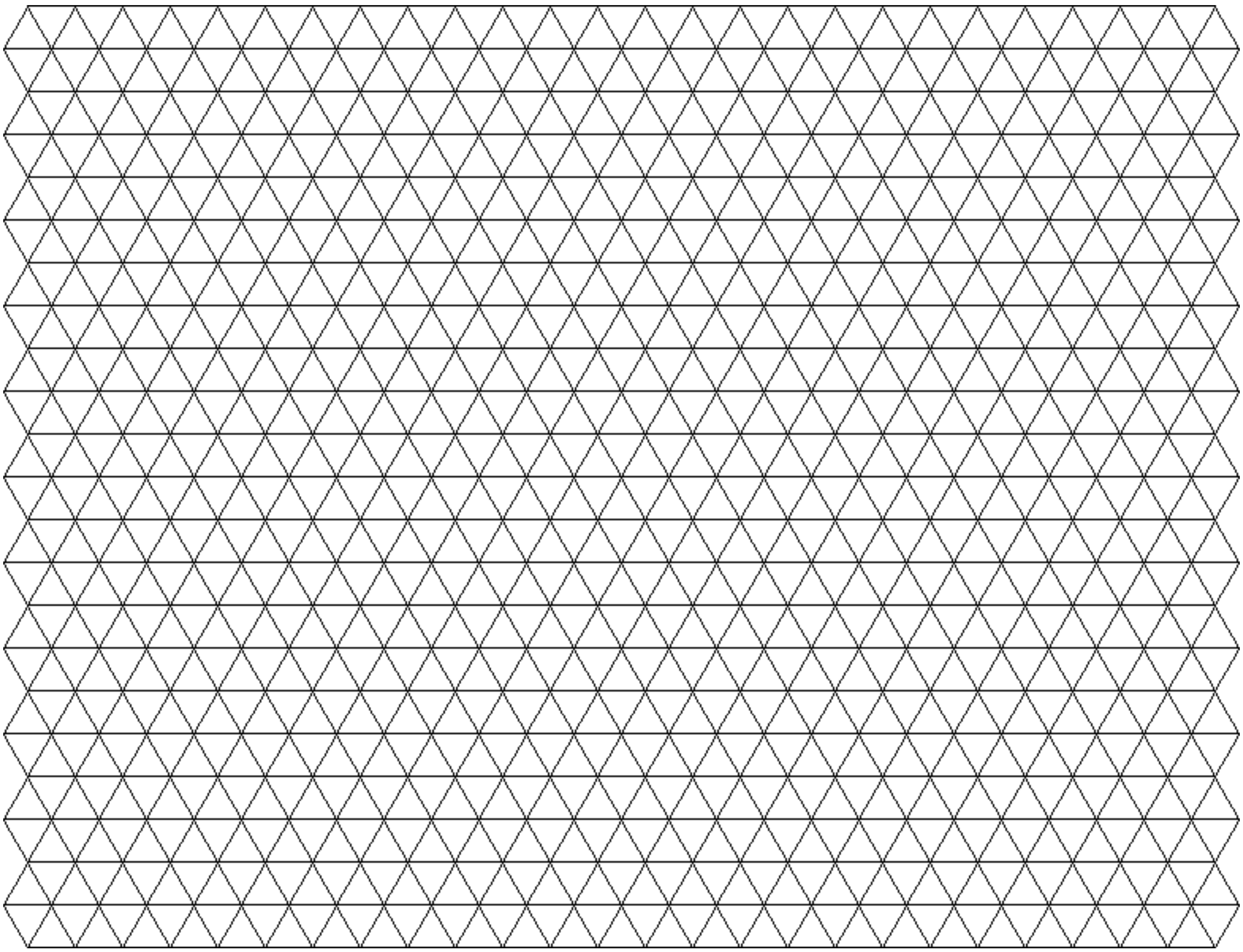
Tessellations are not confined to slides, however. You can also FLIP pieces upside down or side to side. This produces a mirror image. If they are slid apart on the same line in the plane, they make a reflection.

Construct a triangle whose angles are 30, 60, and 90 degrees. Make a cardboard template of this triangle. Can you tessellate on a piece of copier paper by sliding and flipping the triangle?

Try several more triangles with different angle combinations. Does each tessellate by slides and flips?



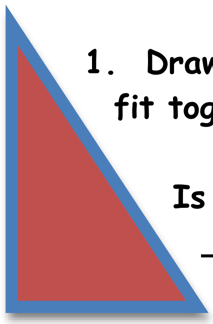




ReplicatorTiles Strike!

These pages and work should go in the Portfolio.

You will need several sheets of cardboard or thick paper, scissors, ruler, and a pencil.



1. Draw one triangle, and then cut out four copies of it. Will your 4 triangles fit together to form a larger triangle? ___________

   Is the larger triangle SIMILAR to one of the smaller triangles? __________

   (Measure the angles and sides of both the larger triangle and one of the smaller triangles to make sure they are similar.)

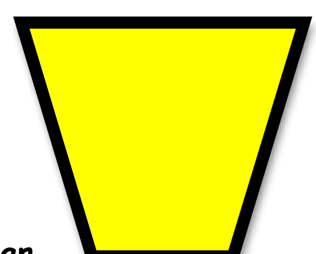
2. Now, trace around the LARGE triangle you made. Cut out 4 copies of this and make an even bigger triangle if you can. Can you use all the triangles? _______

3. Compare your results with each other's. Did it matter what shape the beginning triangle was? _____________

4. These triangles are called a Rep-tile.

SAVE YOUR TRIANGLES.

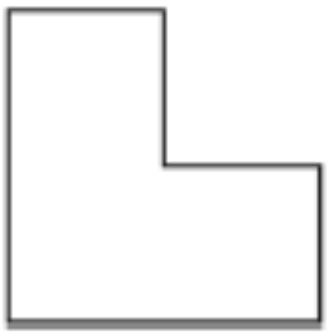
5. Now draw a parallelogram. Make four copies of your parallelogram and see if the four copies will make a parallelogram that is similar to the first one. Do they? ___________

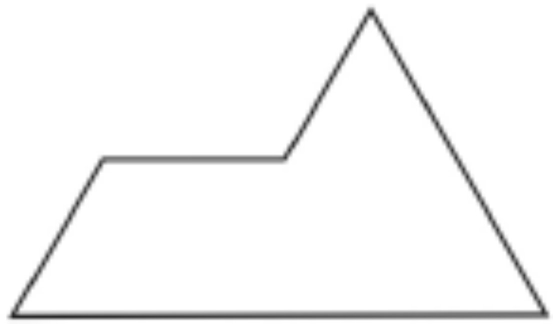


6. Any triangle and any parallelogram is called a rep-4 tile. This means that four copies of a triangle or a parallelogram create a larger similar triangle or parallelogram. Each rep-4 tile is also a rep-9 tile. This means you can use 9 of these tiles to create a larger, similar figure. Make additional copies of either your triangle or parallelogram, and prove this.

   Did you? ___________

7. Below are four other figures. Choose one and make 4 copies. Decide if it makes a rep-4 tile. Compare results with other students. Which one(s) are a rep-4 tile? ____________





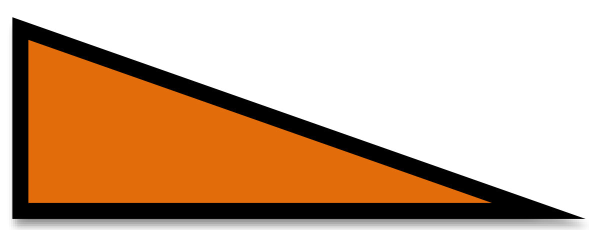
#1 **#2**

#3 **#4**

8. For any natural number n greater than 1, a rep-n tile exists. For instance, a rep-2 tile, a rep-3 tile, rep-5 tiles and so on. Draw an isosceles right triangle, and make copies. Can you discover what rep-n tile it is? ____________

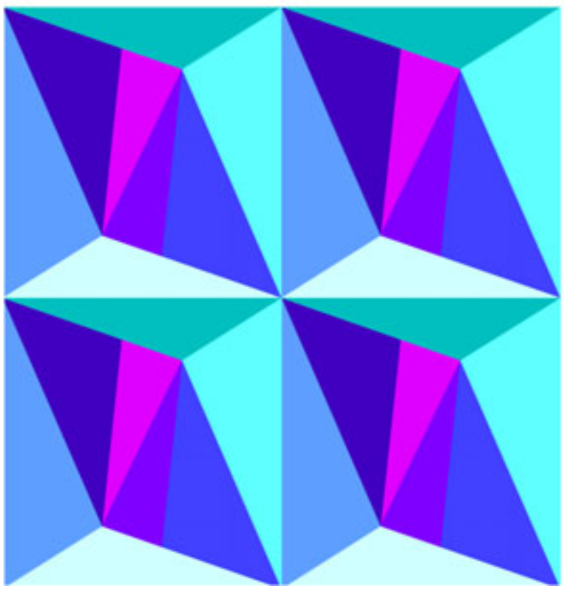
9. Prove that a 30-60-90 triangle is a rep-3 tile. ____________

10. Construct a triangle whose legs are 4 cm. and 2 cm. Make copies and find out what rep-n tile it is. ____________



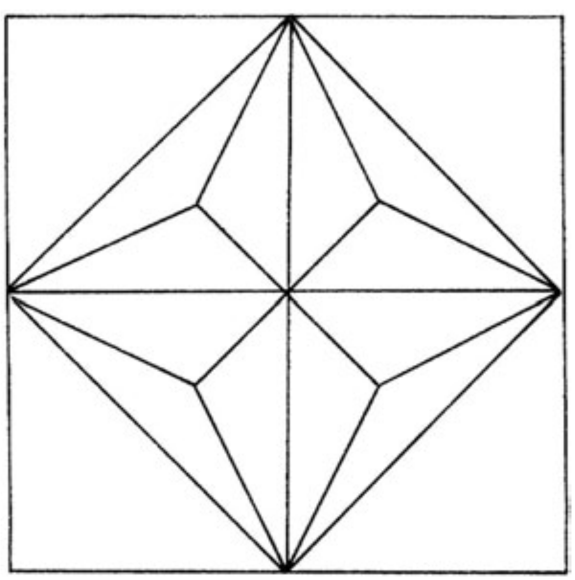
Tessellating With Irregular Polygons

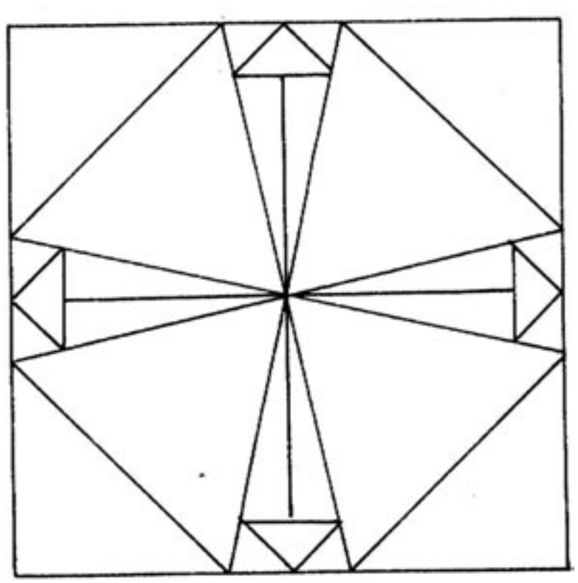
It is fairly easy to see how the regular polygons tessellate. Each angle and side is the same in each shape. But what about irregular ones? In this lesson we will briefly look at some irregular triangles and quadrilaterals.



How many different shaped triangles are repeated in this tessellation? At first glance, because of the colors, it looks like there are 8. However, if you look closer, you will see that there are really only 4 shapes, flipped and translated to form a rectangle, and then that pattern is repeated. In fact if you look closely, you can see parallelograms, rectangles, triangles in triangles, all of these optical patterns formed with color.

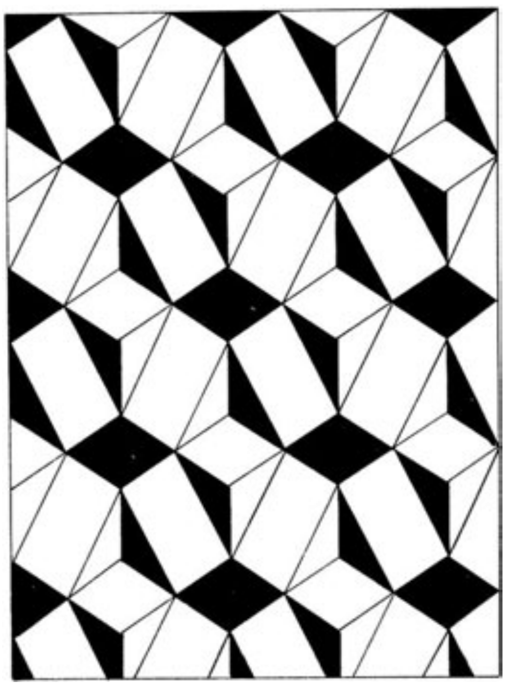
Here are some other patterns that use more than one irregular triangle. Add color to make each look different. Add to Portfolio.

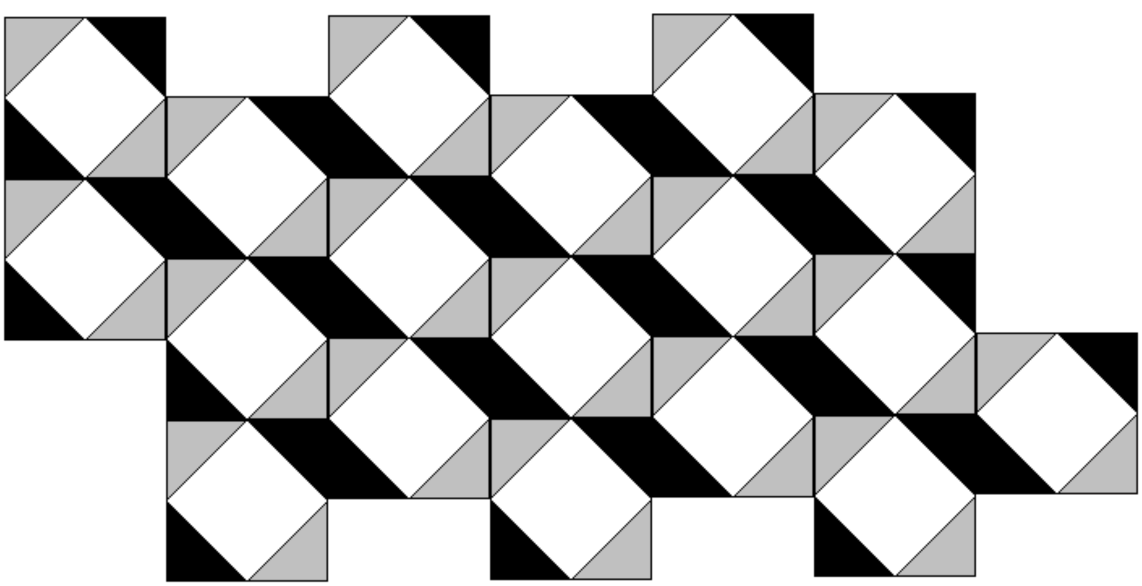




Create at least 2 different irregular triangles. Use them to tessellate a plane area. Add to portfolio.

These examples use more than one shape. There are quite a few of them. Create a tessellation with 2 or more polygons. Add to the Portfolio.



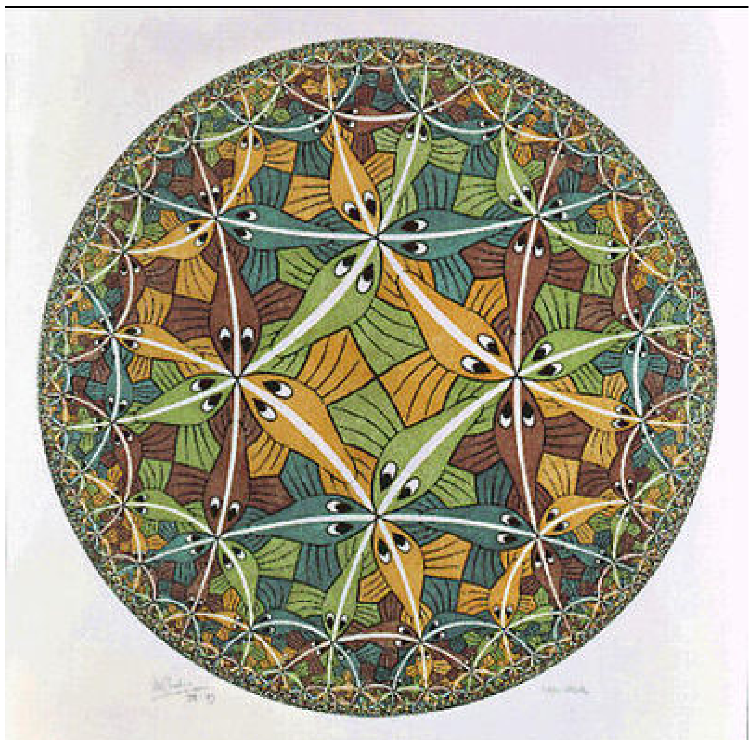


Who Was M.C. Escher?

Please show the PowerPoint "M.C. Esher's Tessellations."

Maurits Cornelis Escher was born in the Dutch province of Friesland in June of 1898. He was the youngest of three sons. In secondary school he had very poor grades. The only thing he was good at was drawing. His art teacher helped his drawing talent and taught him how to make linocuts. He failed his final exams in school and never did graduate.

By 1918 he began private lessons and studies in architecture and was even allowed out of military service to continue his studies. But his health was poor, and he

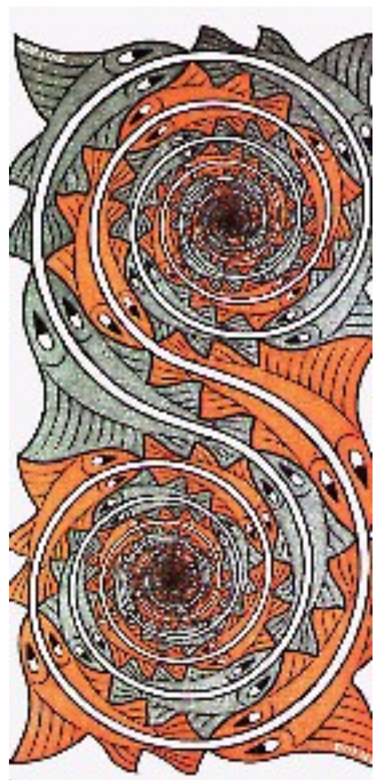


by EC Escher, from Wikipedia Commons

could not keep up. During this time he did many drawings and woodcuts, and people began to get interested in his work.

Although he wanted a career in architecture, after a week at School for Architecture and Arts, he met the artist Samuel de Mesquita. Mesquita and the school's director advised Escher to continue his artwork. He began full time study of the decorative and graphic arts. His landlady also gave him a pet white cat.

Eventually Escher and some friends visited other countries, including Italy and Spain. There he studied the paintings in museums, and travelled around the countryside looking for inspiration. In Granada, Spain, Escher visited the Alhambra and saw examples of Arabic decorative styles. He was very impressed and copied some designs.



By 1923 and 1924 Escher was holding his first one-man shows. He fell in love and married his wife Jetta and went to live in Italy. He continued with woodcuts and prints and lithographs. Several children were born, and the family moved around from Italy to France to Holland to Germany to Switzerland.

In 1930 he met his lifelong friend Bas Kist. Kist was interested in printing techniques, and probably encouraged his friend to do is first linoleum cut works.

By 1937, Escher began to turn away from the outside world, and look inside for vision. This was caused partly by the brewing war in Europe (WWII). As he experimented with shapes, his brother was impressed with the potential applications of his work to crystallography in geology. Escher continued to experiment with shapes, transformations, tilings, and plane-fliling techniques. One of these techniques was tessellations.

Tessellations repeat the same shape or shapes over and over again. Many quilt and floor tile patterns are tessellations. Escher applied this concept to his art and changed the regular polygon shapes to resemble lizards, horses, birds, and bugs. He used the mathematical techniques of symmetry and translation. Some of his works are pure tessellations, others are blends of other perspective techniques.

His father died in 1939, and in 1940 the Nazi invasion of Holland and Belgium kept him from attending his mother's funeral. The Nazi persecution of the Jews also affected Escher in a personal way. His old teacher and mentor de Mesquita was Jewish. He was taken away in 1944 and killed. Escher helped to rescue some of de Mesquita's work to a museum in Amsterdam. He kept one sketch for himself – a sketch that had the imprint of a Nazi boot on it – for the rest of his life.

After the war ended he became quite popular, and designed ceilings, tapestries, and other designs as well as his prints. His fame in America came in the fifties as his work was featured in magazines.

Mathematicians had been appreciating the math in Escher's art since the 1940s, and the end of the 1950s say Escher in demand at meetings of mathematicians and geological crystallographers. He published several successful books about his work that were translated into several languages.

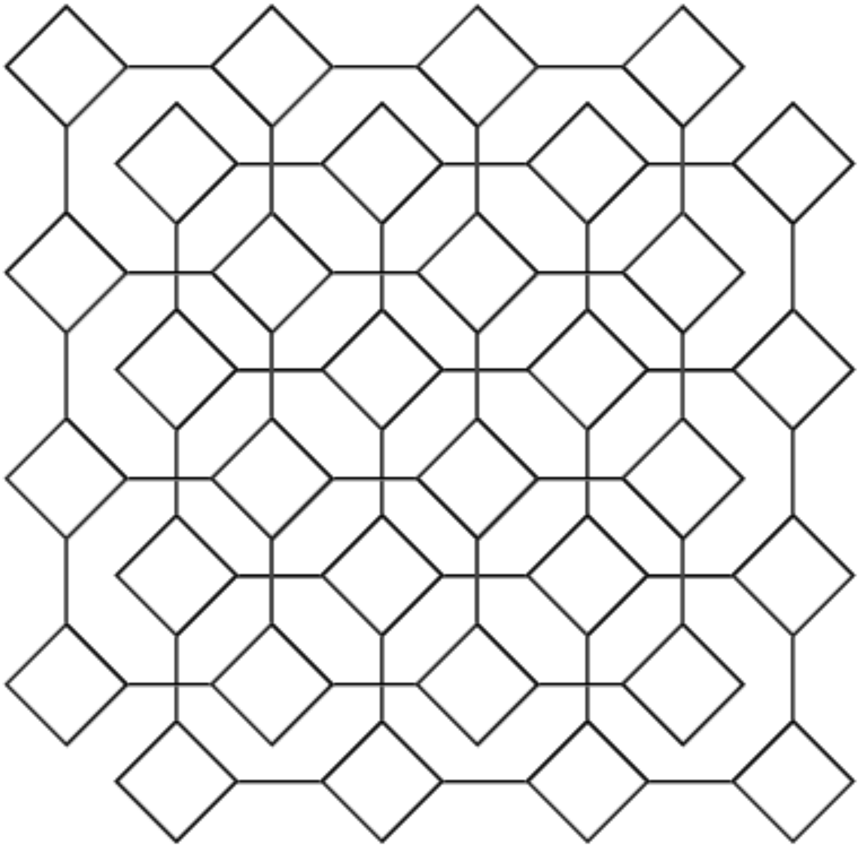
Escher's work continues to be analyzed by artists, designers, and mathematicians. Most people find his tessellations delightful.

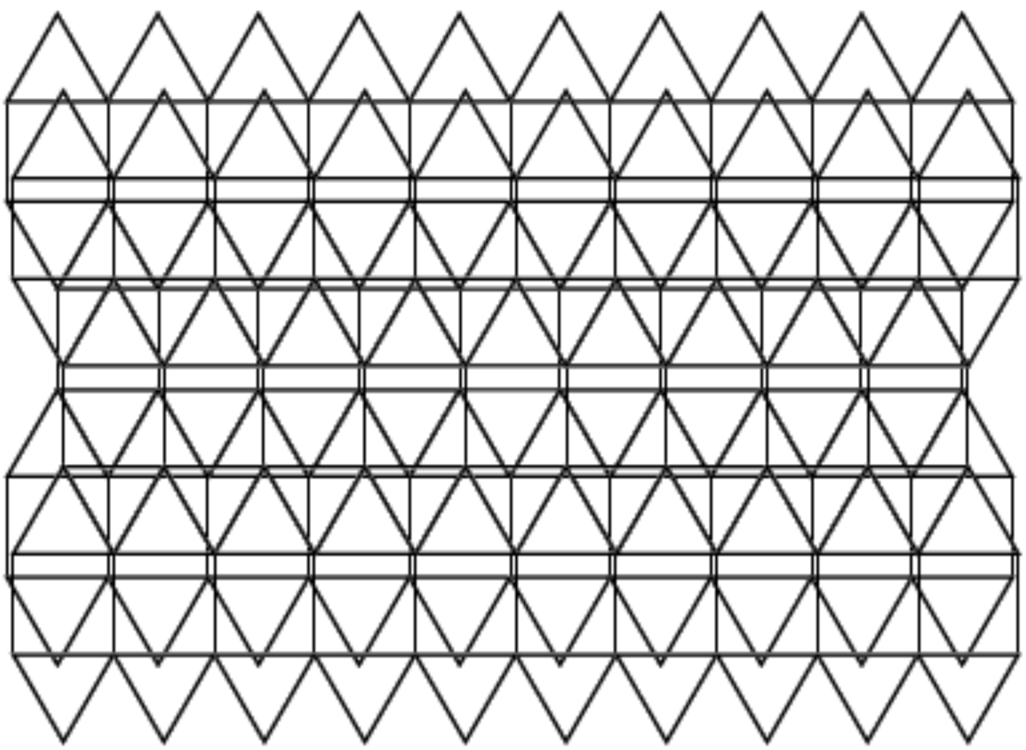




JOB

Islamic Patterns



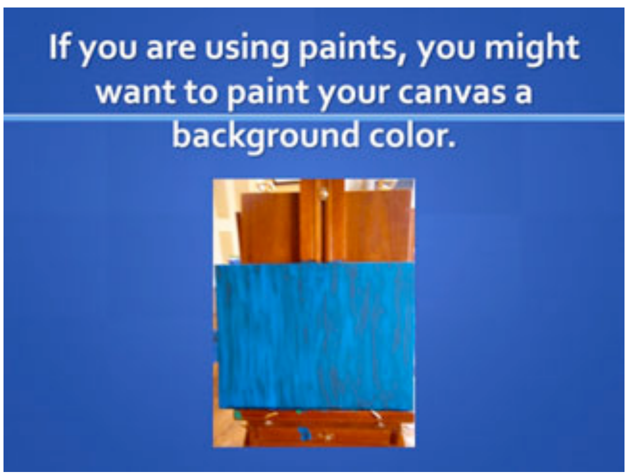


Your Own Escher Tessellation

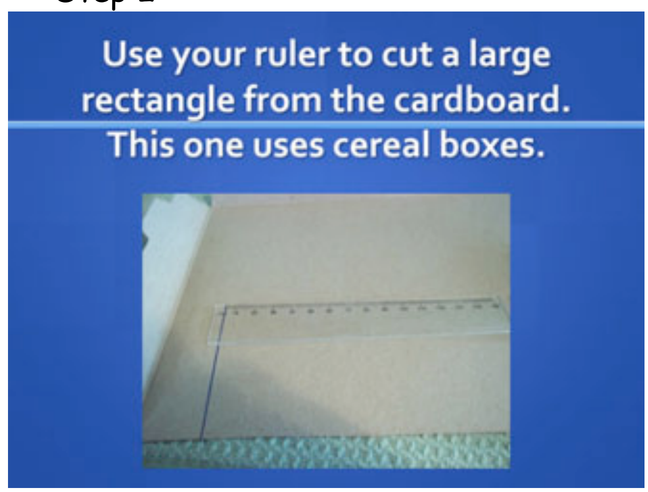
This is also included as a PowerPoint of the same name.

Step 1

Step 2

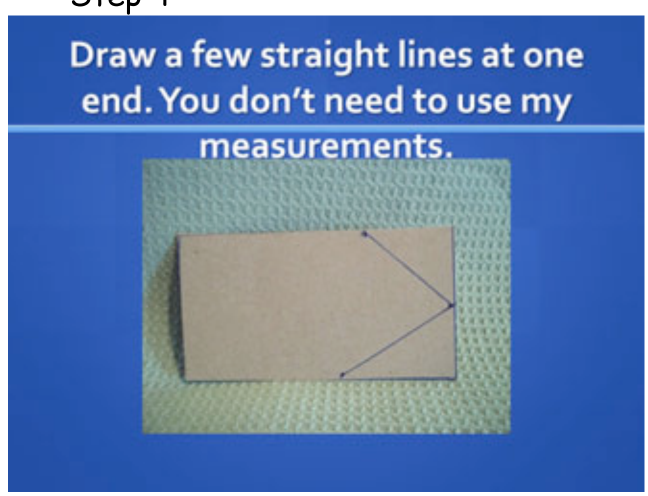


Step 3



Step 4

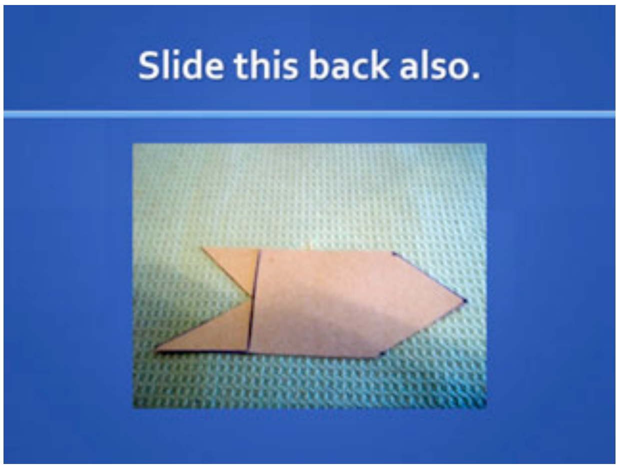
Step 5



Step 6

Step 7

Step 8



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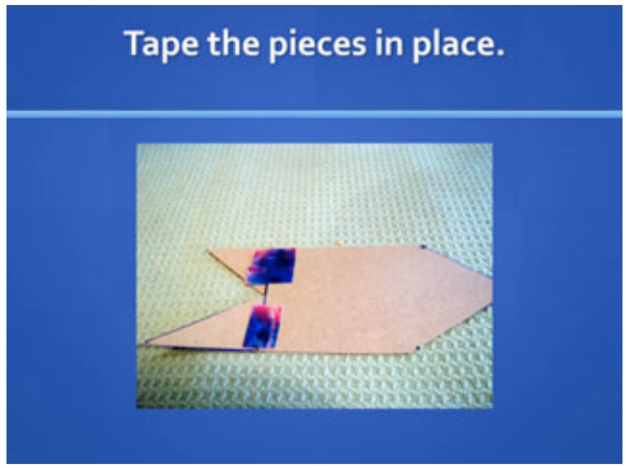

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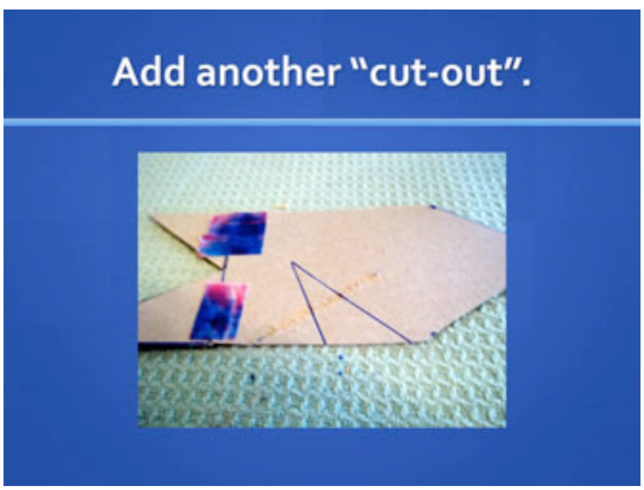

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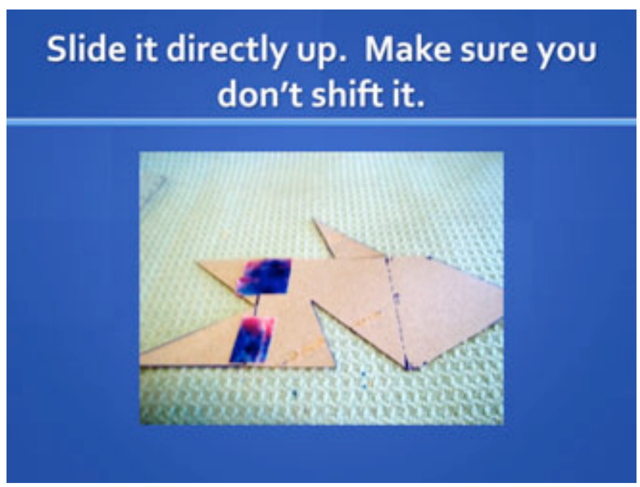
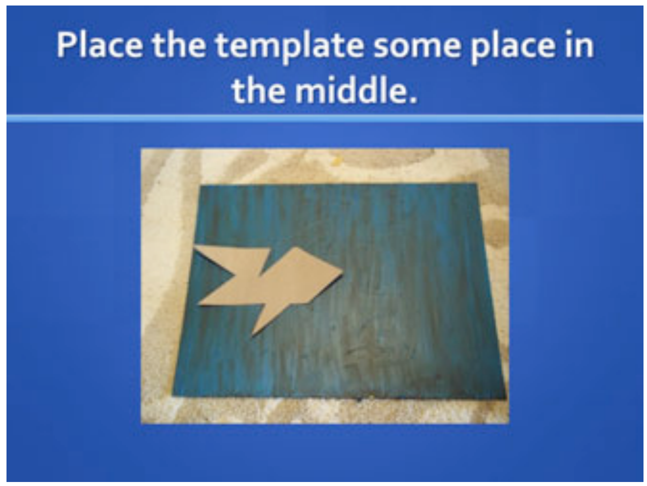


Step 12

Step 13

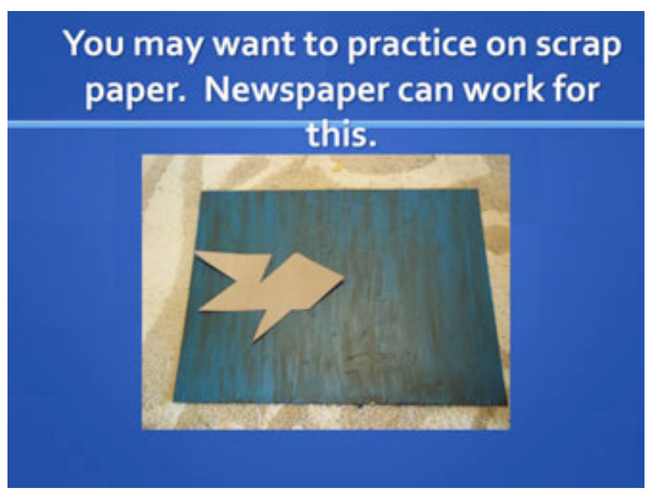
Step 14

Step 15



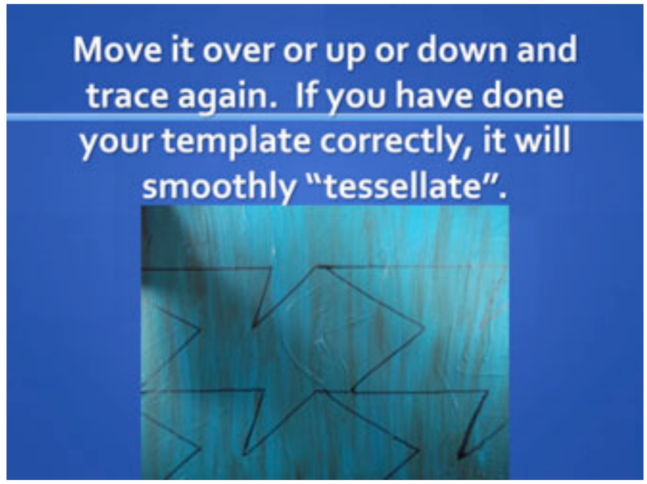
Step 16

Step 17



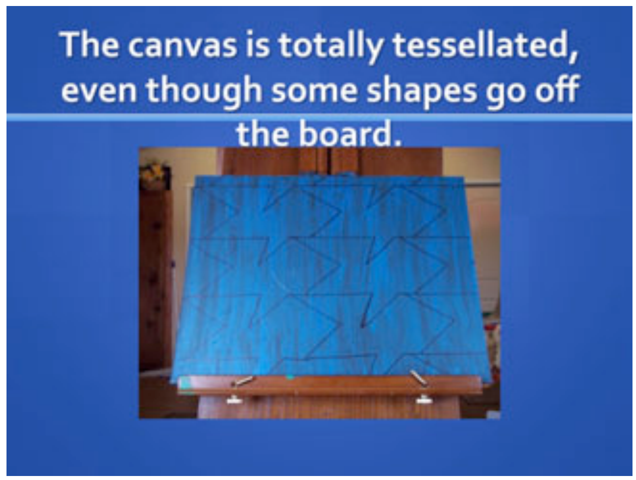
Step 18

Step 19



Step 20

Step 21



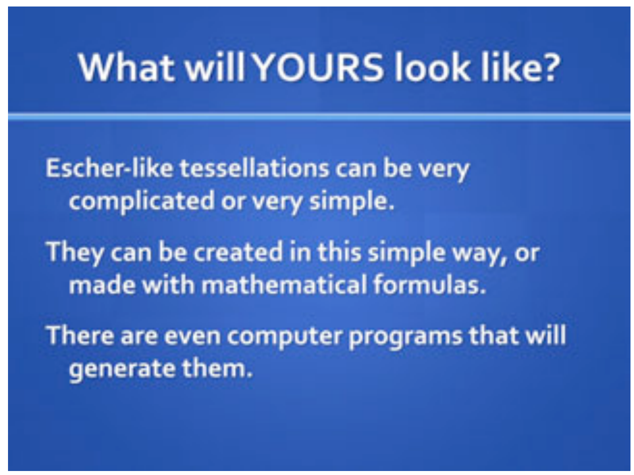
Step 22

Step 23

Step 24

Step26

Step 25

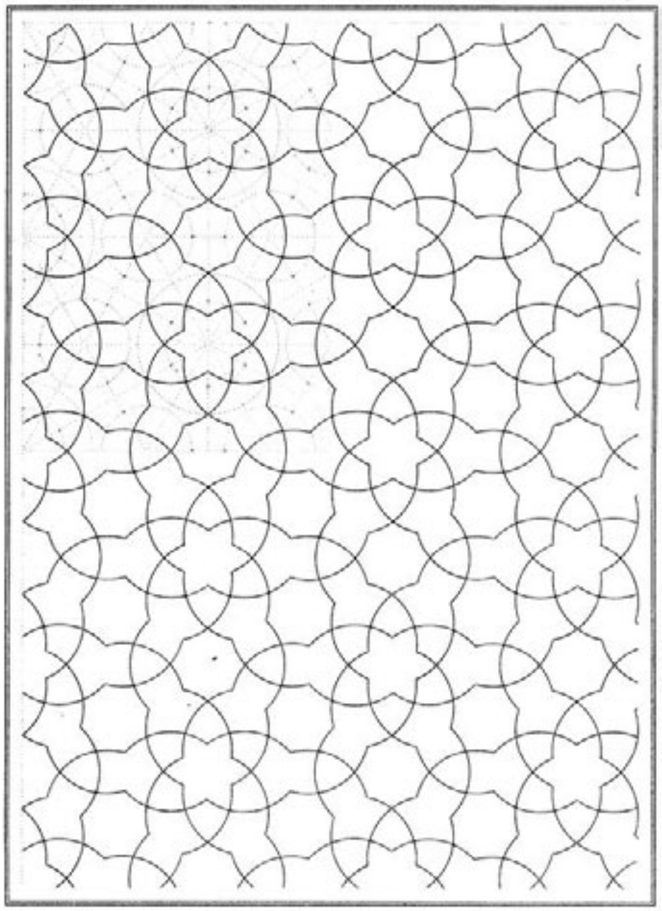


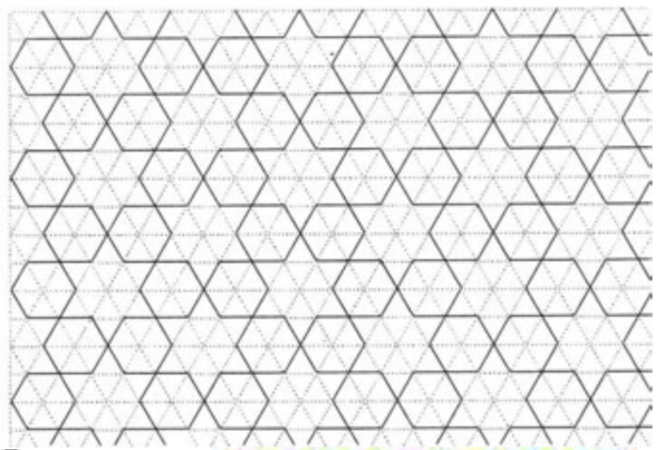
Step 27

Step 28

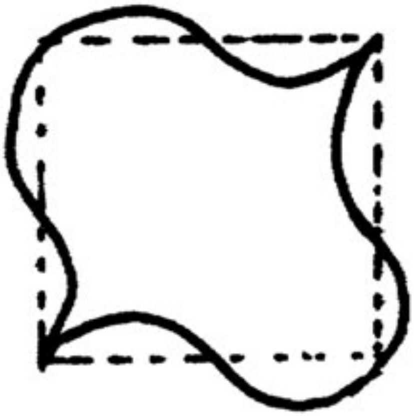
Irregular Shapes and Tessellations

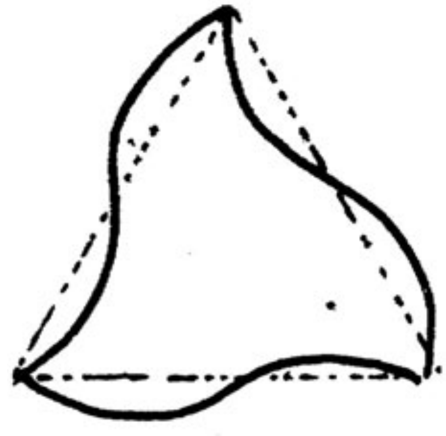
Watch the m4v Escher Tessellations

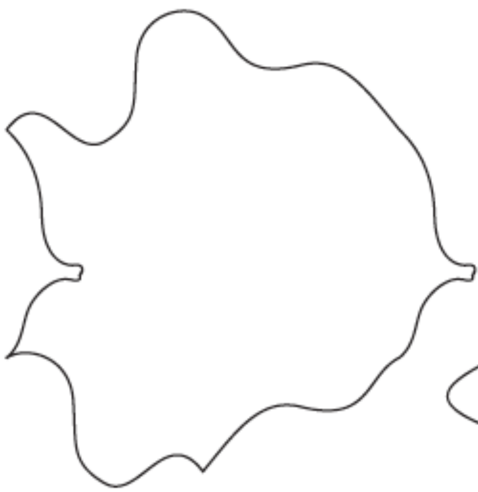


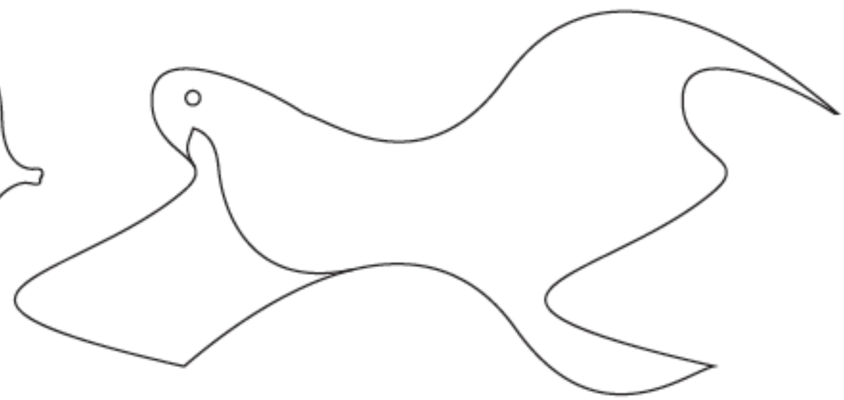


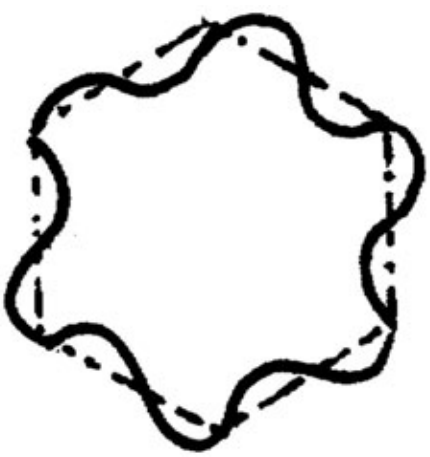
Templates

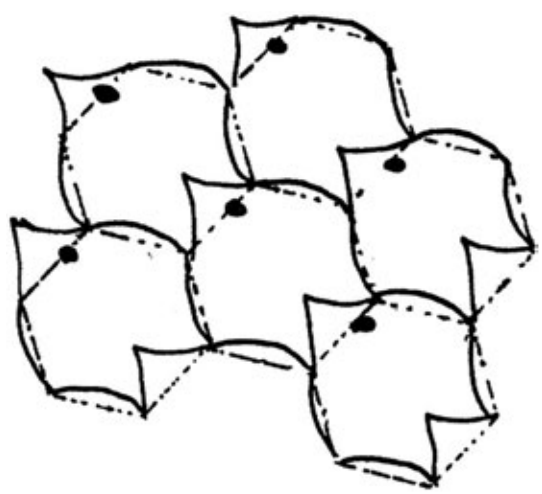




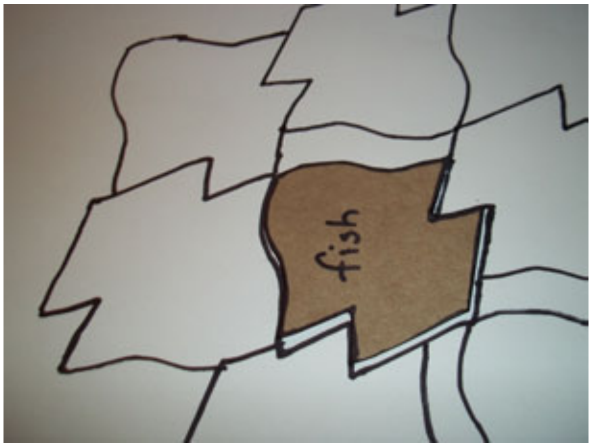








See the next page for ways to use the templates.



Portfolio: Create a tessellation using one of these templates or creating your own template.

Your Company Tiles a Floor

(This activity is available as a WebQuest.)

Repeated patterns on a floor have been around a long, long time. Daniel of the Bible would have stood on a mosaic floor and seen many mosaics on the walls of ancient Babylon.

These wonderful mosaics come from Babylon. More than likely, Daniel from Bible times would have seen these, perhaps even watching them be created.

Today, we use many different patterns in our flooring, fabrics, wall coverings, and so on. But somebody has to design these patterns. This is going to be YOUR job.

Materials needed:

This project can be done several different ways. One is by using pattern blocks or construction paper. Another is by using graph paper. You can also design one using a tessellation program online. You may also need glue, scissors and tape.

Your assignment:

To design a tile pattern for a floor, and then estimate the costs for tiling a 5 x 8 foot floor.

Choose a partner.

FLAREGROUP

Give your company a name and a logo.

Find some clipart to use as a logo or design one. (This is not meant to be the focus of this assignment, so PLEASE do not spend a lot of time on this part.) Make a letterhead for your company using this design.

ImageNewcastle

Design a floor tile.

The tile should be square.

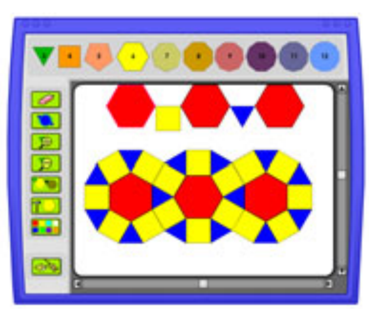
It should be a tessellated design.

It should use a minimum of 3 colors.

It should have a minimum of 2 polygon shapes .

Make four of them.

Cut out your 4 tiles and tape them together on the back so that we can see how the floor will look.



This program can be used to create and print tiles. Find it at Tessellations Creator:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=202

Go to a website and get 3 prices for a 12 by 12 floor tile. Average these three prices (record work on a piece of paper to show later) to get a price for your tile. Go to one of these sites and type in vinyl tile.

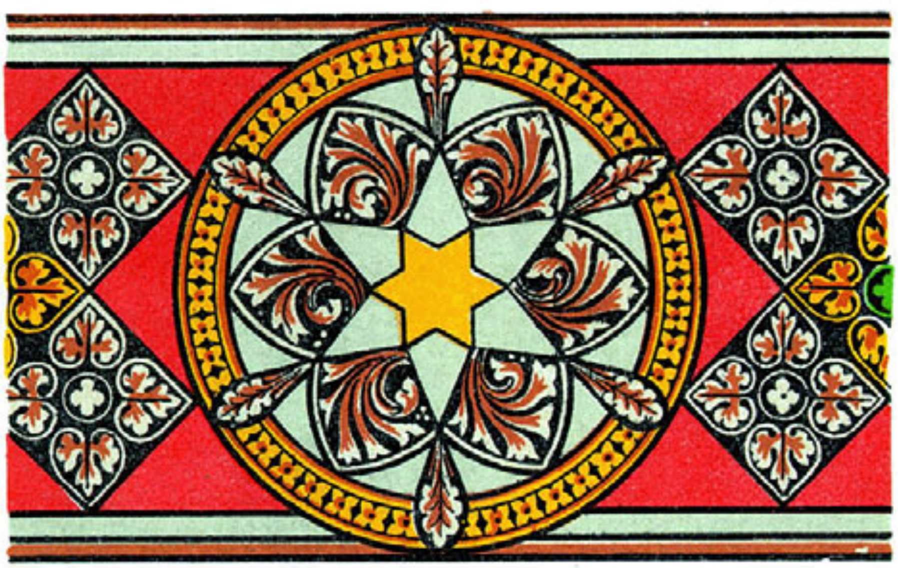
Lowe's: www.**lowes**.com/ Home Depot: www.**homedepot**.com/

Figure out how many tiles you will need to cover the 5 x 8 floor. Your tile will be sold in boxes, not by the piece.

Assume that each box of tile has 15 tiles in it. How many boxes will you need? Will there be leftovers? Remember, you have to buy full boxes. There are also taxes (use 7%).

You will charge $5 a tile to lay the tiles. This fee includes the mortar you need to lay them.

Give your customer an estimate. Paperclip this estimate done on your company letterhead to your tile pattern and post your bid on the board.



WORKSHEET

Your Name: ______________

Three prices for floor tile: ______________
____
____

Average price per tile: ______________

Square footage of 5' x 8' floor: ______________

Number of tiles needed: ______________

Number of boxes need if there are 15 tiles in each box: _____________

Cost of a box of tiles at your average tile cost: ________________

Total cost of tiles you need to buy: __________________________

Cost to install tiles @ $5 a tile (only for tiles that are needed): __________

Subtotal: Cost of tiles & installation: ____________________

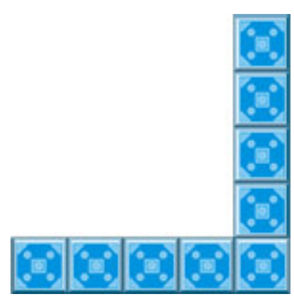
7% sales tax on the subtotal: __________________

Total cost: Add the subtotal and sales tax: _______________

EVALUATION SHEET

Your job bid should have the following:

- Your company name
- Your company logo
- Partners in your company
- Date
- Your estimate
- Attach the tile sample
- Attach this worksheet



Additional Projects:
Finding Math In The World

Quilts

Investigate the different quilt patterns that show tessellation and symmetry. Choose five of your favorites and lay them out on graph paper. Then reproduce one square of each either with fabric or art media. Many of the patterns make beautiful paintings that can be displayed. Try making them miniature!

Cars

Many of the features on a car such as hubcaps and grills show symmetry and tessellations. Make a collection of at least 20 pictures of car parts that illustrate these things. Include radial symmetry and tessellation patterns. Then design your own grill or hubcap.

Pennsylvania Dutch Hex Signs

The Pennsylvania Dutch often decorate their barns and sheds with colorful patterns that show radial and reflective symmetry. Find five pictures of different signs, and then design and paint (or color) one of your own.

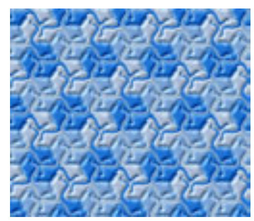
Resources on the Web

M. C. Escher: The Official Website
http://www.mcescher.com/

On this website you can find information about the use of M.C. Escher's work, a short biography, news, bibliography, links and some fun stuff like a Virtual Ride through some of his works. Many know M.C. Escher only for his mathematical prints, but in fact he made much more wonderful art during his lifetime. Recently added six GALLERIES of selected works of art by M.C. Escher to the website.

Tessellations.org
http://www.tessellations.org/
This site is really detailed, includes directions for making tessellations with a whole gallery of art from all ages. See what other students and adults have done.

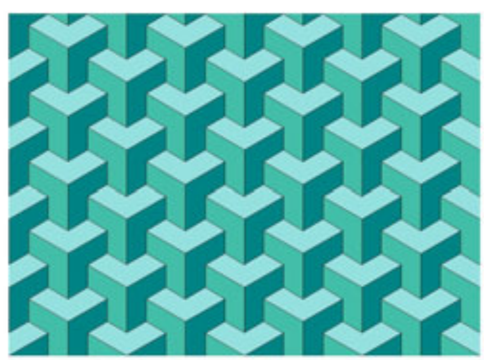
ORACLEThinkQuest
http://library.thinkquest.org/16661/
Although a few of the links need updating, this site has very clear and easy information about tessellations and M.C. Esher's art.



MathSalamanders
http://www.math-salamanders.com/printable-geometry-worksheets.html
http://www.math-salamanders.com/shapes-clip-art.html

Easy to use site with free downloadable and printable math sheets that are very simply made and often one concept. Great resource place.

- Geometry Sheets
- Fraction Sheets
- Money Sheets
- Math Games
- Math Fact Sheets
- Number Grids
- Math Videos
- Math Support Pages



Illuminations

http://illuminations.nctm.org/

The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring mathematics learning of the highest quality for all students. NCTM is the world's largest organization dedicated to improving math education in preK-12.

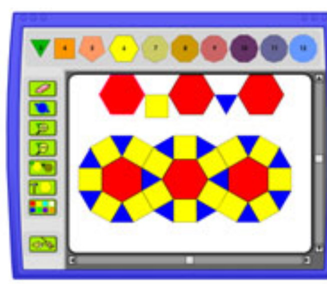
Illuminations is designed to:

- Provide Standards-based resources that improve the teaching and learning of mathematics for all students.
- Provide materials that illuminate the vision for school mathematics set forth in *Principles and Standards for School Mathematics*, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, and *Focus in High School Mathematics: Reasoning and Sense Making*.

Dynamic Paper

http://illuminations.nctm.org/ActivityDetail.aspx?ID=205

Need a pentagonal pyramid that's six inches tall? Or a number line that goes from -18 to 32 by 5's? Or a set of pattern blocks where all shapes have one-inch sides? You can create all those things and more with the Dynamic Paper tool. Place the images you want, then export it as a PDF activity sheet for your students or as a JPEG image for use in other applications or on the web.



Mirror Tool

http://illuminations.nctm.org/ActivityDetail.aspx?ID=24

This tool is used to experiment with symmetry.

Tessellations Creator

http://illuminations.nctm.org/ActivityDetail.aspx?ID=202

The Dynamic Paper tool from Illuminations will also make tessellations that can be saved in PDF and then saved. This tool adds color and allows lots of intricate to scale tessellations to be made.

Thinkfinity

http://www.thinkfinity.org/

This is the VerizonFoundation's prestigious Verizon Thinkfinity website with just tons of amazing high quality lesson plans, games, summer activities, homework, and after school activities. Well worth checking out.

Print Resources

Introduction to TESSELLATIONS, by Dale Seymour and Jill Britton, Dale Seymour Publications, USA, 1989.

M.C.Escher: The Graphic Work, published by Barnes & Noble, Inc.,USA, by arrangement with TASCHEN GmbH, 2007

Symmetry and Tessellations Investigating Patterns, Jill Britton, Dale Seymour Publications, USA, 2000